7. The Linear Factorization Theorem states that an nth-degree polynomial can be expressed as the product of a nonzero constant and ____ linear factors, where each linear factor has a leading coefficient of ____.

Use Descartes's Rule of Signs to determine whether each statement is true or false.

8. A polynomial function with four sign changes must have four positive real zeros. ____

9. A polynomial function with one sign change must have one positive real zero. ____

10. A polynomial function with seven sign changes can have one, three, five, or seven positive real zeros. ____

EXERCISE SET 2.5

Practice Exercises

In Exercises 1–8, use the Rational Zero Theorem to list all possible rational zeros for each given function.

1. $f(x) = x^3 + x^2 - 4x - 4$

2. $f(x) = x^3 + 3x^2 - 6x - 8$

3. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

4. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

5. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

6. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

7. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$

8. $f(x) = 4x^5 - 8x^4 - x + 2$

In Exercises 9–16,

a. List all possible rational zeros.

b. Use synthetic division to test the possible rational zeros and find an actual zero.

c. Use the quotient from part (b) to find the remaining zeros of the polynomial function.

9. $f(x) = x^3 + x^2 - 4x - 4$

10. $f(x) = x^3 - 2x^2 - 11x + 12$

11. $f(x) = 2x^3 - 3x^2 - 11x + 6$

12. $f(x) = 2x^3 - 5x^2 + x + 2$

13. $f(x) = x^3 + 4x^2 - 3x - 6$

14. $f(x) = 2x^3 + x^2 - 3x + 1$

15. $f(x) = 2x^3 + 6x^2 + 5x + 2$

16. $f(x) = x^3 - 4x^2 + 8x - 5$

In Exercises 17–24,

a. List all possible rational roots.

b. Use synthetic division to test the possible rational roots and find an actual root.

c. Use the quotient from part (b) to find the remaining roots and solve the equation.

17. $x^3 - 2x^2 - 11x + 12 = 0$

18. $x^3 - 2x^2 - 7x - 4 = 0$

19. $x^3 - 10x - 12 = 0$

20. $x^3 - 5x^2 + 17x - 13 = 0$

21. $6x^3 + 25x^2 - 24x + 5 = 0$

22. $2x^3 - 5x^2 - 6x + 4 = 0$

23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$

24. $x^4 - 2x^2 - 16x - 15 = 0$

In Exercises 25–32, find an nth-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, use it to graph the function and verify the real zeros and the given function value.

25. $n = 3$; 1 and $5i$ are zeros; $f(-1) = -104$

26. $n = 3$; 4 and $2i$ are zeros; $f(-1) = -50$

27. $n = 3$; -5 and $4 + 3i$ are zeros; $f(2) = 91$

28. $n = 3$; 6 and $-5 + 2i$ are zeros; $f(2) = -636$

29. $n = 4$; i and $3i$ are zeros; $f(-1) = 20$

30. $n = 4$; -2, $-\frac{1}{2}$, and i are zeros; $f(1) = 18$

31. $n = 4$; -2, 5, and $3 + 2i$ are zeros; $f(1) = -96$

32. $n = 4$; -4, $\frac{1}{3}$, and $2 + 3i$ are zeros; $f(1) = 100$

In Exercises 33–38, use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.

33. $f(x) = x^3 + 2x^2 + 5x + 4$

34. $f(x) = x^3 + 7x^2 + x + 7$

35. $f(x) = 5x^3 - 3x^2 + 3x - 1$

36. $f(x) = -2x^3 + x^2 - x + 7$

37. $f(x) = 2x^4 - 5x^3 - x^2 - 6x + 4$

38. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

In Exercises 39–52, find all zeros of the polynomial function or solve the given polynomial equation. Use the Rational Zero Theorem, Descartes's Rule of Signs, and possibly the graph of the polynomial function shown by a graphing utility as an aid in obtaining the first zero or the first root.

39. $f(x) = x^3 - 4x^2 - 7x + 10$

40. $f(x) = x^3 + 12x^2 + 21x + 10$

41. $2x^3 - x^2 - 9x - 4 = 0$

42. $3x^3 - 8x^2 - 8x + 8 = 0$

43. $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$

44. $f(x) = x^4 - 4x^3 - x^2 + 14x + 10$

45. $x^4 - 3x^3 - 20x^2 - 24x - 8 = 0$

46. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$

47. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

48. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

49. $4x^4 - x^3 + 5x^2 - 2x - 6 = 0$

50. $3x^4 - 11x^3 - 3x^2 - 6x + 8 = 0$

51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$

52. $4x^5 + 12x^4 - 41x^3 - 99x^2 + 10x + 24 = 0$

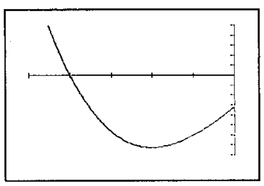
Practice Plus

Exercises 53–60 show incomplete graphs of given polynomial functions.

a. Find all the zeros of each function.

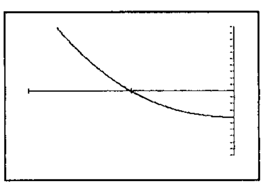
b. Without using a graphing utility, draw a complete graph of the function.

53. $f(x) = -x^3 + x^2 + 16x - 16$



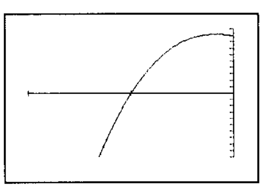
[−5, 0, 1] by [−40, 25, 5]

54. $f(x) = -x^3 + 3x^2 - 4$



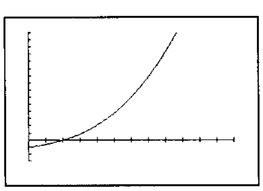
[−2, 0, 1] by [−10, 10, 1]

55. $f(x) = 4x^3 - 8x^2 - 3x + 9$



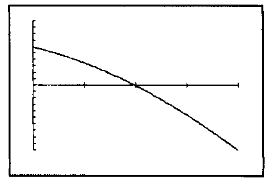
[−2, 0, 1] by [−10, 10, 1]

56. $f(x) = 3x^3 + 2x^2 + 2x - 1$



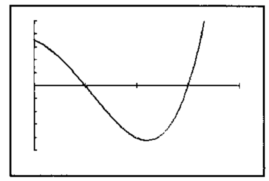
$[0, 2, \frac{1}{6}]$ by [−3, 15, 1]

57. $f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$



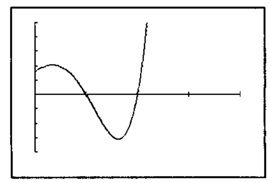
$[0, 1, \frac{1}{4}]$ by [−10, 10, 1]

58. $f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$



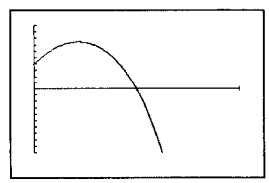
[0, 4, 1] by [−50, 50, 10]

59. $f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$



[0, 4, 1] by [−20, 25, 5]

60. $f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$



[0, 2, 1] by [−10, 10, 1]

Application Exercises

A popular model of carry-on luggage has a length that is 10 inches greater than its depth. Airline regulations require that the sum of the length, width, and depth cannot exceed 40 inches. These conditions, with the assumption that this sum is 40 inches, can be modeled by a function that gives the volume of the luggage, V, in cubic inches, in terms of its depth, x, in inches.

Volume	=	depth	·	length	·	width: 40 − (depth + length)
$V(x)$	=	x	·	$(x + 10)$	·	$[40 - (x + x + 10)]$
$V(x)$	=	$x(x + 10)(30 - 2x)$				

Use function V to solve Exercises 61–62.

61. If the volume of the carry-on luggage is 2000 cubic inches, determine two possibilities for its depth. Where necessary, round to the nearest tenth of an inch.

62. If the volume of the carry-on luggage is 1500 cubic inches, determine two possibilities for its depth. Where necessary, round to the nearest tenth of an inch.