ACHIEVING SUCCESS

Take some time to consider how you are doing in this course. Check your performance by answering the following questions:

- Are you attending all lectures?
- For each hour of class time, are you spending at least two hours outside of class completing all homework assignments, checking answers, correcting errors, and using all resources to get the help that you need?
- Are you reviewing for quizzes and tests?
- Are you reading the textbook? In all college courses, you are responsible for the information in the text, whether or not it is covered in class.
- Are you keeping an organized notebook? Does each page have the appropriate section number from the text on top? Do the pages contain examples your instructor works during lecture and other relevant class notes? Have you included your worked-out homework exercises? Do you keep a special section for graded exams?
- Are you analyzing your mistakes and learning from your errors?
- Are there ways you can improve how you are doing in the course?

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. The degree of the polynomial function $f(x) = -2x^3(x - 1)(x + 5)$ is ______. The leading coefficient is ______.

2. True or false: Some polynomial functions of degree 2 or higher have breaks in their graphs. ______

3. The behavior of the graph of a polynomial function to the far left or the far right is called its ______ behavior, which depends upon the ______ term.

4. The graph of $f(x) = x^3$ ______ to the left and ______ to the right.

5. The graph of $f(x) = -x^3$ ______ to the left and ______ to the right.

6. The graph of $f(x) = x^2$ ______ to the left and ______ to the right.

7. The graph of $f(x) = -x^2$ ______ to the left and ______ to the right.

8. True or false: Odd-degree polynomial functions have graphs with opposite behavior at each end. ______

9. True or false: Even-degree polynomial functions have graphs with the same behavior at each end. ______

10. Every real zero of a polynomial function appears as a/an ______ of the graph.

11. If r is a zero of even multiplicity, then the graph touches the x-axis and ______ at r. If r is a zero of odd multiplicity, then the graph ______ the x-axis at r.

12. If f is a polynomial function and $f(a)$ and $f(b)$ have opposite signs, then there must be at least one value of c between a and b for which $f(c) =$ ______. This result is called the ______ Theorem.

13. If f is a polynomial function of degree n, then the graph of f has at most ______ turning points.

EXERCISE SET 2.3

Practice Exercises

In Exercises 1–10, determine which functions are polynomial functions. For those that are, identify the degree.

1. $f(x) = 5x^2 + 6x^3$

2. $f(x) = 7x^2 + 9x^4$

3. $g(x) = 7x^5 - \pi x^3 + \frac{1}{5}x$

4. $g(x) = 6x^7 + \pi x^5 + \frac{2}{3}x$

5. $h(x) = 7x^3 + 2x^2 + \frac{1}{x}$

6. $h(x) = 8x^3 - x^2 + \frac{2}{x}$

7. $f(x) = x^{\frac{1}{2}} - 3x^2 + 5$

8. $f(x) = x^{\frac{1}{3}} - 4x^2 + 7$

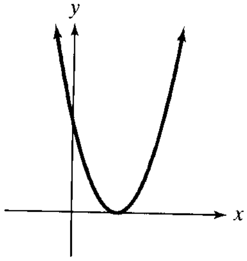
9. $f(x) = \dfrac{x^2 + 7}{x^3}$

10. $f(x) = \dfrac{x^2 + 7}{3}$

See Answers to Selected Exercises.

In Exercises 11–14, identify which graphs are not those of polynomial functions. 12. and 13. are not polynomial functions.

11.



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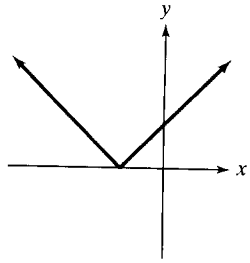

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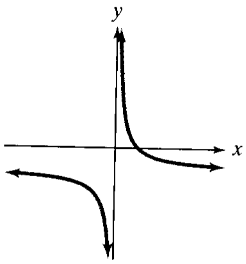

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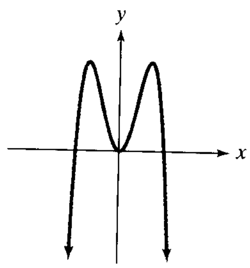


In Exercises 15–18, use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph. [The graphs are labeled (a) through (d).]

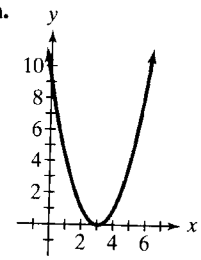
15. $f(x) = -x^4 + x^2$

16. $f(x) = x^3 - 4x^2$

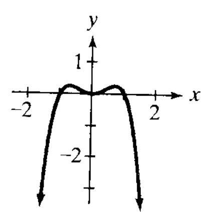
17. $f(x) = (x - 3)^2$

18. $f(x) = -x^3 - x^2 + 5x - 3$

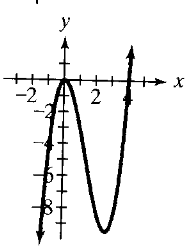
a.



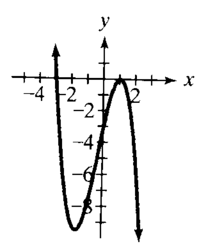
b.



c.



d.



In Exercises 19–24, use the Leading Coefficient Test to determine the end behavior of the graph of the polynomial function.

19. $f(x) = 5x^3 + 7x^2 - x + 9$

20. $f(x) = 11x^3 - 6x^2 + x + 3$

21. $f(x) = 5x^4 + 7x^2 - x + 9$

22. $f(x) = 11x^4 - 6x^2 + x + 3$

23. $f(x) = -5x^4 + 7x^2 - x + 9$

24. $f(x) = -11x^4 - 6x^2 + x + 3$

In Exercises 25–32, find the zeros for each polynomial function and give the multiplicity for each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

25. $f(x) = 2(x - 5)(x + 4)^2$

26. $f(x) = 3(x + 5)(x + 2)^2$

27. $f(x) = 4(x - 3)(x + 6)^3$

28. $f(x) = -3\left(x + \frac{1}{2}\right)(x - 4)^3$

29. $f(x) = x^3 - 2x^2 + x$

30. $f(x) = x^3 + 4x^2 + 4x$

31. $f(x) = x^3 + 7x^2 - 4x - 28$

32. $f(x) = x^3 + 5x^2 - 9x - 45$

In Exercises 33–40, use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers.

33. $f(x) = x^3 - x - 1$; between 1 and 2

34. $f(x) = x^3 - 4x^2 + 2$; between 0 and 1

35. $f(x) = 2x^4 - 4x^2 + 1$; between −1 and 0

36. $f(x) = x^4 + 6x^3 - 18x^2$; between 2 and 3

37. $f(x) = x^3 + x^2 - 2x + 1$; between −3 and −2

38. $f(x) = x^5 - x^3 - 1$; between 1 and 2

39. $f(x) = 3x^3 - 10x + 9$; between −3 and −2

40. $f(x) = 3x^3 - 8x^2 + x + 2$; between 2 and 3

In Exercises 41–64,

a. Use the Leading Coefficient Test to determine the graph's end behavior.

b. Find the x-intercepts. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each intercept.

c. Find the y-intercept.

d. Determine whether the graph has y-axis symmetry, origin symmetry, or neither.

e. If necessary, find a few additional points and graph the function. Use the maximum number of turning points to check whether it is drawn correctly.

41. $f(x) = x^3 + 2x^2 - x - 2$

42. $f(x) = x^3 + x^2 - 4x - 4$

43. $f(x) = x^4 - 9x^2$

44. $f(x) = x^4 - x^2$

45. $f(x) = -x^4 + 16x^2$

46. $f(x) = -x^4 + 4x^2$

47. $f(x) = x^4 - 2x^3 + x^2$

48. $f(x) = x^4 - 6x^3 + 9x^2$

49. $f(x) = -2x^4 + 4x^3$

50. $f(x) = -2x^4 + 2x^3$

51. $f(x) = 6x^3 - 9x - x^5$

52. $f(x) = 6x - x^3 - x^5$

53. $f(x) = 3x^2 - x^3$

54. $f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. $f(x) = -3(x - 1)^2(x^2 - 4)$

56. $f(x) = -2(x - 4)^2(x^2 - 25)$

57. $f(x) = x^2(x - 1)^3(x + 2)$

58. $f(x) = x^3(x + 2)^2(x + 1)$

59. $f(x) = -x^2(x - 1)(x + 3)$

60. $f(x) = -x^2(x + 2)(x - 2)$

61. $f(x) = -2x^3(x - 1)^2(x + 5)$

62. $f(x) = -3x^3(x - 1)^2(x + 3)$

63. $f(x) = (x - 2)^2(x + 4)(x - 1)$

64. $f(x) = (x + 3)(x + 1)^3(x + 4)$