Check Point 12 Solve the equation, correct to four decimal places, for $0 \le x < 2\pi$:

$$\cos^2 x + 5\cos x + 3 = 0.$$

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. The solutions of $\sin x = \frac{\sqrt{2}}{2}$ in $[0, 2\pi)$ are $x = \frac{\pi}{4}$ and $x =$ ______. If n is any integer, all solutions of $\sin x = \frac{\sqrt{2}}{2}$ are given by $x =$ ______ and $x =$ ______.
2. The solution of $\tan x = -\sqrt{3}$ in $[0, \pi)$ is $x = \pi - \frac{\pi}{3}$, or $x =$ ______. If n is any integer, all solutions of $\tan x = -\sqrt{3}$ are given by ______.
3. True or false: If $3x = \frac{\pi}{4} + n\pi$ for any integer n, then $x = \frac{\pi}{12} + n\pi$. ______
4. True or false: If $\cos\frac{x}{3} = \frac{1}{2}$, then $\frac{x}{3} = \frac{\pi}{3} + 2n\pi$ or $\frac{x}{3} = \frac{5\pi}{3} + 2n\pi$ for any integer n. ______
5. True or false: If $\tan 3x = 1$, then $x = \frac{\pi}{4} + n\pi$ for any integer n. ______
6. If $2\cos^2 x - 9\cos x - 5 = 0$, then ______ $= 0$ or ______ $= 0$. Of these two equations, the equation that has no solution is ______.
7. If $2\sin x\cos x + \sqrt{2}\cos x = 0$, then ______ $= 0$ or ______ $= 0$.
8. The first step in solving the equation $4\cos^2 x + 4\sin x - 5 = 0, 0 \le x < 2\pi$, is to replace ______ with ______.
9. If $\sin 0.9695 \approx 0.8246$, then the solutions of $\sin x = -0.8246, 0 \le x < 2\pi$, are given by $x \approx$ ______ $+ 0.9695$ and $x \approx$ ______ $- 0.9695$.

EXERCISE SET 6.5

Practice Exercises

In Exercises 1–10, use substitution to determine whether the given x-value is a solution of the equation.

1. $\cos x = \frac{\sqrt{2}}{2}, \quad x = \frac{\pi}{4}$
2. $\tan x = \sqrt{3}, \quad x = \frac{\pi}{3}$
3. $\sin x = \frac{\sqrt{3}}{2}, \quad x = \frac{\pi}{6}$
4. $\sin x = \frac{\sqrt{2}}{2}, \quad x = \frac{\pi}{3}$
5. $\cos x = -\frac{1}{2}, \quad x = \frac{2\pi}{3}$
6. $\cos x = -\frac{1}{2}, \quad x = \frac{4\pi}{3}$
7. $\tan 2x = -\frac{\sqrt{3}}{3}, \quad x = \frac{5\pi}{12}$
8. $\cos\frac{2x}{3} = -\frac{1}{2}, \quad x = \pi$
9. $\cos x = \sin 2x, \quad x = \frac{\pi}{3}$
10. $\cos x + 2 = \sqrt{3}\sin x, \quad x = \frac{\pi}{6}$

In Exercises 11–24, find all solutions of each equation.

11. $\sin x = \frac{\sqrt{3}}{2}$
12. $\cos x = \frac{\sqrt{3}}{2}$
13. $\tan x = 1$
14. $\tan x = \sqrt{3}$
15. $\cos x = -\frac{1}{2}$
16. $\sin x = -\frac{\sqrt{2}}{2}$
17. $\tan x = 0$
18. $\sin x = 0$
19. $2\cos x + \sqrt{3} = 0$
20. $2\sin x + \sqrt{3} = 0$
21. $4\sin\theta - 1 = 2\sin\theta$
22. $5\sin\theta + 1 = 3\sin\theta$
23. $3\sin\theta + 5 = -2\sin\theta$
24. $7\cos\theta + 9 = -2\cos\theta$

Exercises 25–38 involve equations with multiple angles. Solve each equation on the interval $[0, 2\pi)$.

25. $\sin 2x = \frac{\sqrt{3}}{2}$
26. $\cos 2x = \frac{\sqrt{2}}{2}$
27. $\cos 4x = -\frac{\sqrt{3}}{2}$
28. $\sin 4x = -\frac{\sqrt{2}}{2}$
29. $\tan 3x = \frac{\sqrt{3}}{3}$
30. $\tan 3x = \sqrt{3}$
31. $\tan\frac{x}{2} = \sqrt{3}$
32. $\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$
33. $\sin\frac{2\theta}{3} = -1$
34. $\cos\frac{2\theta}{3} = -1$
35. $\sec\frac{3\theta}{2} = -2$
36. $\cot\frac{3\theta}{2} = -\sqrt{3}$
37. $\sin\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$
38. $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Exercises 39–52 involve trigonometric equations quadratic in form. Solve each equation on the interval $[0, 2\pi)$.

39. $2\sin^2 x - \sin x - 1 = 0$
40. $2\sin^2 x + \sin x - 1 = 0$
41. $2\cos^2 x + 3\cos x + 1 = 0$
42. $\cos^2 x + 2\cos x - 3 = 0$
43. $2\sin^2 x = \sin x + 3$
44. $2\sin^2 x = 4\sin x + 6$

45. $\sin^2 \theta - 1 = 0$
46. $\cos^2 \theta - 1 = 0$
47. $4 \cos^2 x - 1 = 0$
48. $4 \sin^2 x - 3 = 0$
49. $9 \tan^2 x - 3 = 0$
50. $3 \tan^2 x - 9 = 0$
51. $\sec^2 x - 2 = 0$
52. $4 \sec^2 x - 2 = 0$

In Exercises 53–62, solve each equation on the interval $[0, 2\pi)$.

53. $(\tan x - 1)(\cos x + 1) = 0$
54. $(\tan x + 1)(\sin x - 1) = 0$
55. $(2 \cos x + \sqrt{3})(2 \sin x + 1) = 0$
56. $(2 \cos x - \sqrt{3})(2 \sin x - 1) = 0$
57. $\cot x(\tan x - 1) = 0$
58. $\cot x(\tan x + 1) = 0$
59. $\sin x + 2 \sin x \cos x = 0$
60. $\cos x - 2 \sin x \cos x = 0$
61. $\tan^2 x \cos x = \tan^2 x$
62. $\cot^2 x \sin x = \cot^2 x$

In Exercises 63–84, use an identity to solve each equation on the interval $[0, 2\pi)$.

63. $2 \cos^2 x + \sin x - 1 = 0$
64. $2 \cos^2 x - \sin x - 1 = 0$
65. $\sin^2 x - 2 \cos x - 2 = 0$
66. $4 \sin^2 x + 4 \cos x - 5 = 0$
67. $4 \cos^2 x = 5 - 4 \sin x$
68. $3 \cos^2 x = \sin^2 x$
69. $\sin 2x = \cos x$
70. $\sin 2x = \sin x$
71. $\cos 2x = \cos x$
72. $\cos 2x = \sin x$
73. $\cos 2x + 5 \cos x + 3 = 0$
74. $\cos 2x + \cos x + 1 = 0$
75. $\sin x \cos x = \frac{\sqrt{2}}{4}$
76. $\sin x \cos x = \frac{\sqrt{3}}{4}$
77. $\sin x + \cos x = 1$
78. $\sin x + \cos x = -1$
79. $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$
80. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$
81. $\sin 2x \cos x + \cos 2x \sin x = \frac{\sqrt{2}}{2}$
82. $\sin 3x \cos 2x + \cos 3x \sin 2x = 1$
83. $\tan x + \sec x = 1$
84. $\tan x - \sec x = 1$

In Exercises 85–96, use a calculator to solve each equation, correct to four decimal places, on the interval $[0, 2\pi)$.

85. $\sin x = 0.8246$
86. $\sin x = 0.7392$
87. $\cos x = -\frac{2}{5}$
88. $\cos x = -\frac{4}{7}$
89. $\tan x = -3$
90. $\tan x = -5$
91. $\cos^2 x - \cos x - 1 = 0$
92. $3 \cos^2 x - 8 \cos x - 3 = 0$
93. $4 \tan^2 x - 8 \tan x + 3 = 0$
94. $\tan^2 x - 3 \tan x + 1 = 0$
95. $7 \sin^2 x - 1 = 0$
96. $5 \sin^2 x - 1 = 0$

In Exercises 97–116, use the most appropriate method to solve each equation on the interval $[0, 2\pi)$. *Use exact values where possible or give approximate solutions correct to four decimal places.*

97. $2 \cos 2x + 1 = 0$
98. $2 \sin 3x + \sqrt{3} = 0$
99. $\sin 2x + \sin x = 0$
100. $\sin 2x + \cos x = 0$
101. $3 \cos x - 6\sqrt{3} = \cos x - 5\sqrt{3}$
102. $\cos x - 5 = 3 \cos x + 6$
103. $\tan x = -4.7143$
104. $\tan x = -6.2154$
105. $2 \sin^2 x = 3 - \sin x$
106. $2 \sin^2 x = 2 - 3 \sin x$
107. $\cos x \csc x = 2 \cos x$
108. $\tan x \sec x = 2 \tan x$
109. $5 \cot^2 x - 15 = 0$
110. $5 \sec^2 x - 10 = 0$
111. $\cos^2 x + 2 \cos x - 2 = 0$
112. $\cos^2 x + 5 \cos x - 1 = 0$
113. $5 \sin x = 2 \cos^2 x - 4$
114. $7 \cos x = 4 - 2 \sin^2 x$
115. $2 \tan^2 x + 5 \tan x + 3 = 0$
116. $3 \tan^2 x - \tan x - 2 = 0$

Practice Plus

In Exercises 117–120, graph f and g in the same rectangular coordinate system for $0 \le x \le 2\pi$. *Then solve a trigonometric equation to determine points of intersection and identify these points on your graphs.*

117. $f(x) = 3 \cos x, g(x) = \cos x - 1$
118. $f(x) = 3 \sin x, g(x) = \sin x - 1$
119. $f(x) = \cos 2x, g(x) = -2 \sin x$
120. $f(x) = \cos 2x, g(x) = 1 - \sin x$

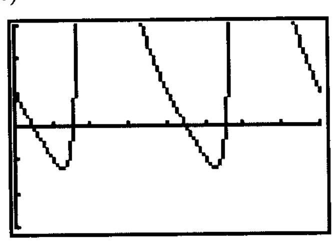
In Exercises 121–126, solve each equation on the interval $[0, 2\pi)$.

121. $|\cos x| = \frac{\sqrt{3}}{2}$
122. $|\sin x| = \frac{1}{2}$
123. $10 \cos^2 x + 3 \sin x - 9 = 0$
124. $3 \cos^2 x - \sin x = \cos^2 x$
125. $2 \cos^3 x + \cos^2 x - 2 \cos x - 1 = 0$ (*Hint:* Use factoring by grouping.)
126. $2 \sin^3 x - \sin^2 x - 2 \sin x + 1 = 0$ (*Hint:* Use factoring by grouping.)

In Exercises 127–128, find the x-intercepts, correct to four decimal places, of the graph of each function. Then use the x-intercepts to match the function with its graph. The graphs are labeled (a) and (b).

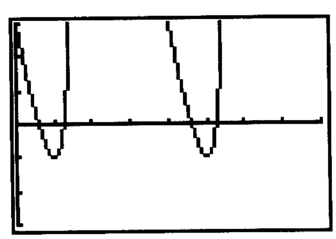
127. $f(x) = \tan^2 x - 3 \tan x + 1$
128. $g(x) = 4 \tan^2 x - 8 \tan x + 3$

a.



$[0, 2\pi, \frac{\pi}{4}]$ by $[-3, 3, 1]$

b.



$[0, 2\pi, \frac{\pi}{4}]$ by $[-3, 3, 1]$