4. $\frac{x}{5}\cdot\frac{x}{3} = \frac{x^2}{15}$

5. $\frac{x}{5}\div\frac{x}{3} = \frac{3}{5}, x \neq 0$

6. $\frac{x^2}{3} - \frac{x-4}{3} = \frac{x^2 - x + 4}{3}$

7. Consider the following subtraction problem:

$$\frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3}.$$

The factors of the first denominator are $(x-2)$ and $(x+3)$.
The factors of the second denominator are $(x+3)$ and $(x+1)$.
The LCD is $(x-2)(x+3)(x+1)$.

8. An equivalent expression for $\frac{3x+2}{x-5}$ with a denominator of $(3x+4)(x-5)$ can be obtained by multiplying the numerator and denominator by $3x+4$.

9. A rational expression whose numerator or denominator or both contain rational expressions is called a/an complex rational expression or a/an complex fraction.

10. $\frac{\frac{1}{x+3}-\frac{1}{x}}{3} = \frac{x(x+3)}{x(x+3)}\cdot\frac{\left(\frac{1}{x+3}-\frac{1}{x}\right)}{3} = \frac{x-(x+3)}{3x(x+3)}$

$$= \frac{-3}{3x(x+3)}$$

$$= \frac{-1}{x(x+3)}$$

11. We can simplify

$$\frac{\sqrt{x}+\frac{1}{\sqrt{x}}}{x}$$

by multiplying the numerator and the denominator by $\sqrt{x}$.

12. We can rationalize the numerator of $\frac{\sqrt{x+7}-\sqrt{x}}{7}$ by multiplying the numerator and the denominator by $\sqrt{x+7}+\sqrt{x}$.

EXERCISE SET P.6

Practice Exercises

In Exercises 1–6, find all numbers that must be excluded from the domain of each rational expression.

1. $\frac{7}{x-3}$ 3
2. $\frac{13}{x+9}$ -9
3. $\frac{x+5}{x^2-25}$ $5, -5$
4. $\frac{x+7}{x^2-49}$ $7, -7$
5. $\frac{x-1}{x^2+11x+10}$ $-1, -10$
6. $\frac{x-3}{x^2+4x-45}$ $9, -5$

In Exercises 7–14, simplify each rational expression. Find all numbers that must be excluded from the domain of the simplified rational expression.

7. $\frac{3x-9}{x^2-6x+9}$ $\frac{3}{x-3}; x \neq 3$
8. $\frac{4x-8}{x^2-4x+4}$
9. $\frac{x^2-12x+36}{4x-24}$ $\frac{x-6}{4}; x \neq 6$
10. $\frac{x^2-8x+16}{3x-12}$
11. $\frac{y^2+7y-18}{y^2-3y+2}$ $\frac{y+9}{y-1}; y \neq 1, 2$
12. $\frac{y^2-4y-5}{y^2+5y+4}$
13. $\frac{x^2+12x+36}{x^2-36}$ $\frac{x+6}{x-6}; x \neq 6, -6$
14. $\frac{x^2-14x+49}{x^2-49}$

In Exercises 15–32, multiply or divide as indicated.

15. $\frac{x-2}{3x+9}\cdot\frac{2x+6}{2x-4}$ $\frac{1}{3}; x \neq 2, -3$
16. $\frac{6x+9}{3x-15}\cdot\frac{x-5}{4x+6}$
17. $\frac{x^2-9}{x^2}\cdot\frac{x^2-3x}{x^2+x-12}$
18. $\frac{x^2-4}{x^2-4x+4}\cdot\frac{2x-4}{x+2}$
19. $\frac{x^2-5x+6}{x^2-2x-3}\cdot\frac{x^2-1}{x^2-4}$ $\frac{x-1}{x-2}; x \neq -1, 2, 3$
20. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$ $\frac{x+3}{x-2}; x \neq -3, -2, 2, 3$
21. $\frac{x^3-8}{x^2-4}\cdot\frac{x+2}{3x}$
22. $\frac{x^2+6x+9}{x^3+27}\cdot\frac{1}{x+3}$
23. $\frac{x+1}{3}\div\frac{3x+3}{7}$ $x \neq -1$
24. $\frac{x+5}{7}\div\frac{4x+20}{9}$
25. $\frac{x^2-4}{x}\div\frac{x+2}{x-2}$
26. $\frac{x^2-4}{x-2}\div\frac{x+2}{4x-8}$
27. $\frac{4x^2+10}{x-3}\div\frac{6x^2+15}{x^2-9}$ $\frac{2(x+3)}{3}; x \neq 3, -3$
28. $\frac{x^2+x}{x^2-4}\div\frac{x^2-1}{x^2+5x+6}$ $\frac{x(x+3)}{(x-2)(x+1)}; x \neq 2, 1, -1, -2, -3$
29. $\frac{x^2-25}{2x-2}\div\frac{x^2+10x+25}{x^2+4x-5}$ $\frac{x-5}{2}; x \neq 1, -5$
30. $\frac{x^2-4}{x^2+3x-10}\div\frac{x^2+5x+6}{x^2+8x+15}$ $1; x \neq 2, -2, -3, -5$
31. $\frac{x^2+x-12}{x^2+x-30}\cdot\frac{x^2+5x+6}{x^2-2x-3}\div\frac{x+3}{x^2+7x+6}$
32. $\frac{x^3-25x}{4x^2}\cdot\frac{2x^2-2}{x^2-6x+5}\div\frac{x^2+5x}{7x+7}$

In Exercises 33–58, add or subtract as indicated.

33. $\frac{4x+1}{6x+5}+\frac{8x+9}{6x+5}$ $2; x \neq -\frac{5}{6}$
34. $\frac{3x+2}{3x+4}+\frac{3x+6}{3x+4}$
35. $\frac{x^2-2x}{x^2+3x}+\frac{x^2+x}{x^2+3x}$ $\frac{2x-1}{x+3}; x \neq 0, -3$

See Answers to Selected Exercises.

36. $\dfrac{x^2-4x}{x^2-x-6}+\dfrac{4x-4}{x^2-x-6}$ $\dfrac{x-2}{x-3}; x\neq -2,3$

37. $\dfrac{4x-10}{x-2}-\dfrac{x-4}{x-2}$ $3; x\neq 2$

38. $\dfrac{2x+3}{3x-6}-\dfrac{3-x}{3x-6}$

39. $\dfrac{x^2+3x}{x^2+x-12}-\dfrac{x^2-12}{x^2+x-12}$ $\dfrac{3}{x-3}; x\neq 3,-4$

40. $\dfrac{x^2-4x}{x^2-x-6}-\dfrac{x-6}{x^2-x-6}$ $\dfrac{x-2}{x+2}; x\neq -2,3$

41. $\dfrac{3}{x+4}+\dfrac{6}{x+5}$

42. $\dfrac{8}{x-2}+\dfrac{2}{x-3}$

43. $\dfrac{3}{x+1}-\dfrac{3}{x}$ $\dfrac{3}{x(x+1)}; x\neq -1,0$

44. $\dfrac{4}{x}-\dfrac{3}{x+3}$

45. $\dfrac{2x}{x+2}+\dfrac{x+2}{x-2}$

46. $\dfrac{3x}{x-3}-\dfrac{x+4}{x+2}$

47. $\dfrac{x+5}{x-5}+\dfrac{x-5}{x+5}$

48. $\dfrac{x+3}{x-3}+\dfrac{x-3}{x+3}$

49. $\dfrac{3}{2x+4}+\dfrac{2}{3x+6}$

50. $\dfrac{5}{2x+8}+\dfrac{7}{3x+12}$

51. $\dfrac{4}{x^2+6x+9}+\dfrac{4}{x+3}$

52. $\dfrac{3}{5x+2}+\dfrac{5x}{25x^2-4}$

53. $\dfrac{3x}{x^2+3x-10}-\dfrac{2x}{x^2+x-6}$

54. $\dfrac{x}{x^2-2x-24}-\dfrac{x}{x^2-7x+6}$

55. $\dfrac{x+3}{x^2-1}-\dfrac{x+2}{x-1}$

56. $\dfrac{x+5}{x^2-4}-\dfrac{x+1}{x-2}$

57. $\dfrac{4x^2+x-6}{x^2+3x+2}-\dfrac{3x}{x+1}+\dfrac{5}{x+2}$ $\dfrac{x-1}{x+2}; x\neq -2,-1$

58. $\dfrac{6x^2+17x-40}{x^2+x-20}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

In Exercises 59–72, simplify each complex rational expression.

59. $\dfrac{\frac{x}{3}-1}{x-3}$ $\dfrac{1}{3}; x\neq 3$

60. $\dfrac{\frac{x}{4}-1}{x-4}$ $\dfrac{1}{4}; x\neq 4$

61. $\dfrac{1+\frac{1}{x}}{3-\frac{1}{x}}$ $\dfrac{x+1}{3x-1}; x\neq 0,\frac{1}{3}$

62. $\dfrac{8+\frac{1}{x}}{4-\frac{1}{x}}$ $\dfrac{8x+1}{4x-1}; x\neq 0,\frac{1}{4}$

63. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x+y}$ $\dfrac{1}{xy}; x\neq 0, y\neq 0, x\neq -y$

64. $\dfrac{1-\frac{1}{x}}{xy}$ $\dfrac{x-1}{xy}; x\neq 0, y\neq 0$

65. $\dfrac{x-\frac{x}{x+3}}{x+2}$ $\dfrac{x}{x+3}; x\neq -2,-3$

66. $\dfrac{x-3}{x-\frac{3}{x-2}}$ $\dfrac{x-2}{x-1}; x\neq 2,3,-1$

67. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}$ $\dfrac{14}{7}; x\neq -2,2$

68. $\dfrac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}$

69. $\dfrac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3}+\frac{1}{x-3}}$ $\dfrac{x-3}{x-2}; x\neq -2,-1,3$

70. $\dfrac{\frac{6}{x^2+2x-15}-\frac{1}{x-3}}{\frac{1}{x+5}+1}$ $\dfrac{x-1}{(x+6)(x-3)}; x\neq -6,-5,3$

71. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

72. $\dfrac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h}$

Exercises 73–78 contain fractional expressions that occur frequently in calculus. Simplify each expression.

73. $\dfrac{\sqrt{x}-\frac{1}{3\sqrt{x}}}{\sqrt{x}}$ $1-\frac{1}{3x}; x>0$

74. $\dfrac{\sqrt{x}-\frac{1}{4\sqrt{x}}}{\sqrt{x}}$ $1-\frac{1}{4x}; x>0$

75. $\dfrac{\frac{x^2}{\sqrt{x^2+2}}-\sqrt{x^2+2}}{x^2}$

76. $\dfrac{\sqrt{5-x^2}+\frac{x^2}{\sqrt{5-x^2}}}{5-x^2}$

77. $\dfrac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$

78. $\dfrac{\frac{1}{\sqrt{x+3}}-\frac{1}{\sqrt{x}}}{3}$

In Exercises 79–82, rationalize the numerator.

79. $\dfrac{\sqrt{x+5}-\sqrt{x}}{5}$ $\dfrac{1}{\sqrt{x+5}+\sqrt{x}}$

80. $\dfrac{\sqrt{x+7}-\sqrt{x}}{7}$ $\dfrac{1}{\sqrt{x+7}+\sqrt{x}}$

81. $\dfrac{\sqrt{x}+\sqrt{y}}{x^2-y^2}$ $\dfrac{1}{(x+y)(\sqrt{x}-\sqrt{y})}$

82. $\dfrac{\sqrt{x}-\sqrt{y}}{x^2-y^2}$ $\dfrac{1}{(x+y)(\sqrt{x}+\sqrt{y})}$

Practice Plus

In Exercises 83–90, perform the indicated operations. Simplify the result, if possible.

83. $\left(\dfrac{2x+3}{x+1}\cdot\dfrac{x^2+4x-5}{2x^2+x-3}\right)-\dfrac{2}{x+2}$ $\dfrac{x^2+5x+8}{(x-2)(x+1)}$

84. $\dfrac{1}{x^2-2x-8}\div\left(\dfrac{1}{x-4}-\dfrac{1}{x+2}\right)$ $\dfrac{1}{6}$

85. $\left(2-\dfrac{6}{x+1}\right)\left(1+\dfrac{3}{x-2}\right)$ 2

86. $\left(4-\dfrac{3}{x+2}\right)\left(1+\dfrac{5}{x-1}\right)$ $\dfrac{(4x+5)(x+4)}{(x+2)(x-1)}$

87. $\dfrac{y^{-1}-(y+5)^{-1}}{5}$ $\dfrac{1}{y(y+5)}$