

1. A farmer has 2400ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?
2. A cylindrical can is to be made to hold 1 liter of oil. Find the dimensions that will minimize the amount of metal required to manufacture the can.

3. Find the point on the parabola  $y^2 = 2x$  that is closest to the point  $(1, 4)$ .

4. A man launches a boat from point A on a bank of a straight river, 3km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly to B, or he could row to some point D between C and B and then run to B. If he can row at 6km/h and run at 8km/h, where should he land to reach B as soon as possible?

5. Find two numbers whose difference is 100 and whose product is a maximum.
6. Find the dimensions of a rectangle with a perimeter 100 meters and whose area is as large as possible.

7. A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs \$10 per square meter, and material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container.

8. A rectangular page is to contain 24 sq. inches of print. The margins at the top and bottom of the page are each 1.5 inches wide. The margins on each side are 1 inch. What should the dimensions of the page be so that the least amount of paper is used.

9. Suppose a manufacturer can sell  $x$  items a week for a revenue of  $r(x) = 200x - .01x^2$  cents and it costs  $c(x) = 50x + 20,000$  cents to make  $x$  items. Is there a most profitable number of items to make each week? Hint: Profit = Revenue - Cost

10. A container in the shape of a right circular cylinder with no top has surface area  $3\pi$  ft.<sup>2</sup>. What height  $h$  and base radius  $r$  will maximize the volume of the cylinder?

11. A cylindrical can is to hold  $20\pi$  m.<sup>3</sup> The material for the top and bottom costs \$10/m.<sup>2</sup> and material for the side costs \$8/m.<sup>2</sup> Find the radius  $r$  and height  $h$  of the most economical can.

12. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of the trees ? What is the maximum output?