SKILLS WARM UP 1.6

The following warm-up exercises involve skills that were covered in a previous course or in earlier sections. You will use these skills in the exercise set for this section. For additional help, review Appendices A.4 and A.5, and Section 1.5.

In Exercises 1–4, simplify the expression.

1. $\dfrac{x^2 + 6x + 8}{x^2 - 6x - 16}$
2. $\dfrac{x^2 - 5x - 6}{x^2 - 9x + 18}$
3. $\dfrac{2x^2 - 2x - 12}{4x^2 - 24x + 36}$
4. $\dfrac{x^3 - 16x}{x^3 + 2x^2 - 8x}$

In Exercises 5–8, solve for x.

5. $x^2 + 7x = 0$
6. $x^2 + 4x - 5 = 0$
7. $3x^2 + 8x + 4 = 0$
8. $3x^3 - x^2 - 24x = 0$

In Exercises 9 and 10, find the limit.

9. $\lim\limits_{x \to 3} (2x^2 - 3x + 4)$
10. $\lim\limits_{x \to -2} \sqrt{x^2 - x + 3}$

Exercises 1.6

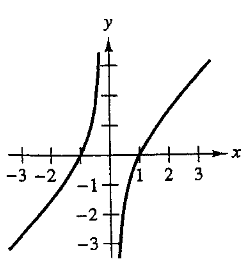
See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Determining Continuity In Exercises 1–10, determine whether the function is continuous on the entire real number line. Explain your reasoning. *See Examples 1 and 2.*

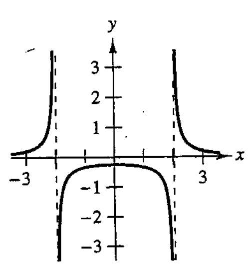
1. $f(x) = 5x^3 - x^2 + 2$
2. $f(x) = (x^2 - 1)^3$
3. $f(x) = \dfrac{3}{x^2 - 16}$
4. $f(x) = \dfrac{1}{9 - x^2}$
5. $f(x) = \dfrac{1}{4 + x^2}$
6. $f(x) = \dfrac{5x}{x^2 + 8}$
7. $f(x) = \dfrac{2x - 1}{x^2 - 8x + 15}$
8. $f(x) = \dfrac{x + 4}{x^2 - 6x + 5}$
9. $g(x) = \dfrac{x^2 - 8x + 12}{x^2 - 36}$
10. $g(x) = \dfrac{x^2 - 11x + 30}{x^2 - 25}$

Determining Continuity In Exercises 11–40, describe the interval(s) on which the function is continuous. Explain why the function is continuous on the interval(s). If the function has a discontinuity, identify the conditions of continuity that are not satisfied. *See Examples 1, 2, 3, 4, and 5.*

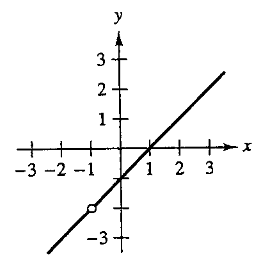
11. $f(x) = \dfrac{x^2 - 1}{x}$



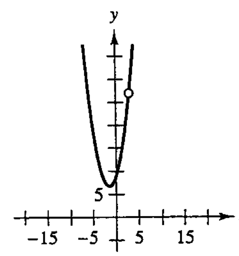
12. $f(x) = \dfrac{1}{x^2 - 4}$



13. $f(x) = \dfrac{x^2 - 1}{x + 1}$

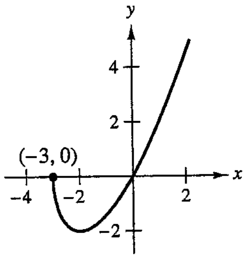


14. $f(x) = \dfrac{x^3 - 27}{x - 3}$

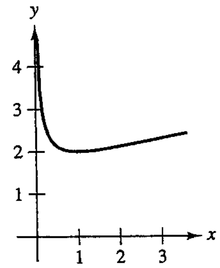


15. $f(x) = x^2 - 9x + 14$
16. $f(x) = 3 - 2x - x^2$
17. $f(x) = \dfrac{x}{x^2 - 1}$
18. $f(x) = \dfrac{x - 3}{x^2 - 9}$
19. $f(x) = \dfrac{7x}{x^2 + 5}$
20. $f(x) = \dfrac{6}{x^2 + 3}$
21. $f(x) = \dfrac{x - 5}{x^2 - 9x + 20}$
22. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$
23. $f(x) = \sqrt{4 - x}$
24. $f(x) = \sqrt{x - 1}$
25. $f(x) = \sqrt{x} + 2$
26. $f(x) = 3 - \sqrt{x}$
27. $f(x) = \begin{cases} -2x + 3, & -1 \le x \le 1 \\ x^2, & 1 < x \le 3 \end{cases}$
28. $f(x) = \begin{cases} \frac{1}{2}x + 1, & -3 \le x \le 2 \\ 3 - x, & 2 < x \le 4 \end{cases}$
29. $f(x) = \begin{cases} 4 - 2x, & x \le 2 \\ x^2 - 3, & x > 2 \end{cases}$
30. $f(x) = \begin{cases} x^2 - 2, & x \le -1 \\ 3x + 2, & x > -1 \end{cases}$
31. $f(x) = \dfrac{|x + 1|}{x + 1}$
32. $f(x) = \dfrac{|4 - x|}{4 - x}$

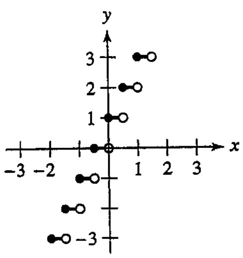
33. $f(x) = x\sqrt{x+3}$



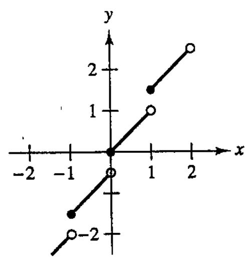
34. $f(x) = \dfrac{x+1}{\sqrt{x}}$



35. $f(x) = [\![2x]\!] + 1$



36. $f(x) = \dfrac{[\![x]\!]}{2} + x$



37. $f(x) = [\![x - 1]\!]$

38. $f(x) = x - [\![x]\!]$

39. $h(x) = f(g(x)),\ \ f(x) = \dfrac{1}{\sqrt{x}},\ \ g(x) = x - 1, x > 1$

40. $h(x) = f(g(x)),\ \ f(x) = \dfrac{1}{x-1},\ \ g(x) = x^2 + 5$

Determining Continuity In Exercises 41–46, sketch the graph of the function and describe the interval(s) on which the function is continuous. If there are any discontinuities, determine whether they are removable.

41. $f(x) = \dfrac{x^2 - 16}{x - 4}$

42. $f(x) = \dfrac{2x^2 + x}{x}$

43. $f(x) = \dfrac{x + 4}{3x^2 - 12}$

44. $f(x) = \dfrac{-x}{x^3 - x}$

45. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \ge 0 \end{cases}$

46. $f(x) = \begin{cases} x^2 - 3, & x \le 0 \\ 2x + 3, & x > 0 \end{cases}$

Determining Continuity on a Closed Interval In Exercises 47–50, discuss the continuity of the function on the closed interval. If there are any discontinuities, determine whether they are removable.

Function	*Interval*
47. $f(x) = x^2 - 4x - 5$	$[-1, 5]$
48. $f(x) = \dfrac{5}{x^2 + 1}$	$[-2, 2]$
49. $f(x) = \dfrac{1}{x - 2}$	$[1, 4]$
50. $f(x) = \dfrac{x - 1}{x^2 - 4x + 3}$	$[0, 4]$

Finding Discontinuities In Exercises 51–56, use a graphing utility to graph the function. Use the graph to determine any x-value(s) at which the function is not continuous. Explain why the function is not continuous at the x-value(s).

51. $h(x) = \dfrac{1}{x^2 - x - 2}$

52. $k(x) = \dfrac{4 - x}{x^2 + x - 12}$

53. $f(x) = \begin{cases} 2x - 4, & x \le 3 \\ x^2 - 2x, & x > 3 \end{cases}$

54. $f(x) = \begin{cases} 3x - 2, & x \le 2 \\ x + 1, & x > 2 \end{cases}$

55. $f(x) = x - 2[\![x]\!]$

56. $f(x) = [\![2x - 1]\!]$

Making a Function Continuous In Exercises 57 and 58, find the constant a (Exercise 57) and the constants a and b (Exercise 58) such that the function is continuous on the entire real number line.

57. $f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$

58. $f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$

Writing In Exercises 59 and 60, use a graphing utility to graph the function on the interval $[-4, 4]$. Does the graph of the function appear to be continuous on this interval? Is the function in fact continuous on $[-4, 4]$? Write a short paragraph about the importance of examining a function analytically as well as graphically.

59. $f(x) = \dfrac{x^2 + x}{x}$

60. $f(x) = \dfrac{x^3 - 8}{x - 2}$

61. Environmental Cost The cost C (in millions of dollars) of removing x percent of the pollutants emitted from the smokestack of a factory can be modeled by

$$C = \frac{2x}{100 - x}.$$

(a) What is the implied domain of C? Explain your reasoning.

(b) Use a graphing utility to graph the cost function. Is the function continuous on its domain? Explain your reasoning.

(c) Find the cost of removing 75% of the pollutants from the smokestack.