

Name: _____

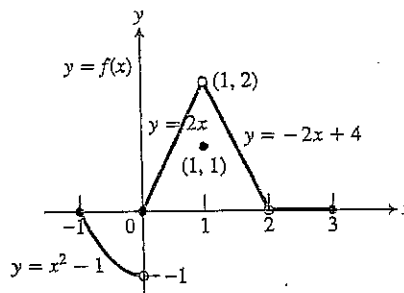
Calculus Honors: Continuity

①

Exercises 2.2

Exercises 1-6 are about the function f defined as follows and whose graph is shown.

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0; \\ 2x, & 0 \leq x < 1; \\ 1, & x = 1; \\ -2x + 4, & 1 < x < 2; \\ 0, & 2 < x \leq 3. \end{cases}$$



- Does $f(-1)$ exist?
 - Does $\lim_{x \rightarrow -1^+} f(x)$ exist?
 - Does $\lim_{x \rightarrow -1^+} f(x) = f(-1)$?
 - Is f continuous at $x = -1$?
- Does $f(1)$ exist?
 - Does $\lim_{x \rightarrow 1} f(x)$ exist?
 - Does $\lim_{x \rightarrow 1} f(x) = f(1)$?
 - Is f continuous at $x = 1$?
- Is f defined at $x = 2$? (Look at the definition of f .)
 - Is f continuous at $x = 2$?
- At what values of x is f continuous?
- What is the value of $\lim_{x \rightarrow 2} f(x)$?
 - Can a function g be defined to make g a continuous extension of f to the point $x = 2$? If so, give g . If not, explain.
- How should h be defined to make h a continuous extension of f to the point $x = 1$?

- The function $f(x)$ is defined by $f(x) = (x^2 - 1)/(x - 1)$ when $x \neq 1$ and by $f(1) = 2$. Is f continuous at $x = 1$? Explain.
- Define $g(3)$ so that $g(x) = (x^2 - 9)/(x - 3)$ is continuous at $x = 3$.
- Define $h(2)$ so that $h(x) = (x^2 + 3x - 10)/(x - 2)$ is continuous at $x = 2$.
- Define $f(1)$ so that $f(x) = (x^3 - 1)/(x^2 - 1)$ is continuous at $x = 1$.
- Define $g(4)$ so that $g(x) = (x^2 - 16)/(x^2 - 3x - 4)$ is continuous at $x = 4$.

$$15. \text{ Let } f(x) = \begin{cases} 0, & x < 0, \\ 1, & 0 \leq x \leq 1, \\ 0, & 1 < x. \end{cases}$$

- Determine a complete graph of f .
- At what points is the function continuous?

$$16. \text{ Let } f(x) = \begin{cases} 1, & x < 0, \\ \sqrt{1 - x^2}, & 0 \leq x \leq 1, \\ x - 1, & x > 1. \end{cases}$$

- Determine a complete graph of f .
- Is f continuous? Explain.

Find the points, if any, at which the functions in Exercises 17-30 are *not* continuous.

$$17. y = \frac{1}{x - 2}$$

$$18. y = \frac{1}{(x + 2)^2}$$

$$19. y = \frac{x + 1}{x^2 - 4x + 3}$$

$$20. y = \frac{x + 3}{x^2 - 3x - 10}$$

$$21. y = \frac{x^3 - 1}{x^2 - 1}$$

$$22. y = \frac{1}{x^2 + 1}$$

$$23. y = |x - 1|$$

$$24. y = |2x + 3|$$

$$25. y = \frac{\cos x}{x}$$

$$26. y = \frac{|x|}{x}$$

$$27. y = \sqrt{2x + 3}$$

$$28. y = \sqrt[3]{3x - 1}$$

$$29. y = \sqrt[3]{2x - 1}$$

$$30. y = \sqrt{2 - x}$$