Name _____ Optimization 1 Date _____ (1)

Guidelines for Solving Optimization Problems

- Read the problem carefully. As you read it, think about what is given and what you are being asked to find. Re-read the problem several times as you continue to think about what is known and what is unknown.
- If possible, sketch and label a diagram using given values and variables as appropriate.
- Formulate any equations or formulas that will establish the relationships between/among the variables.
- Decide which variable is to be maximized or minimized and make certain that this variable is defined in terms of the other variables.
- Find the critical number(s) of the function by taking the derivative, setting it equal to zero, and solving.
- Re-read the problem to determine if you have completely solved the problem.

1. A long rectangular sheet of metal, 12 in. wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up in order to give the gutter its greatest capacity?

2. An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 in. wide and 21 in. long by cutting a square from each corner and then bending up the resulting sides. Find the size of the corner square that will produce a box having the largest possible volume.

3. A circular cylindrical metal container, open at the top, is to have a capacity of 24π in^3. The cost of the material used for the bottom of the container is 15 cents per in^2 and that of the material used for the sides is 5 cents per in^2. If there is no waste of material, find the dimensions that will minimize the cost of the material.

4. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with the maximum volume?

5. The product of two positive numbers is 288. Minimize the sum of twice the first number plus the second.

6. A rectangular page is to contain 24 square inches of print. The margins on each side are 1 inch and at the top and the bottom of the page there are 1½ inch margins. What should the dimensions of the page be so that the least amount of paper is used?

7. Find the dimensions of a rectangular field of maximum area that can be enclosed on all four sides using 200 meters of fencing material.

8. A rectangular field is to be enclosed with a fence. One side of the field is against an existing fence, so that no fence is needed on that side. If material for the fence costs \$2 per foot for the 2 ends and \$4 per foot for the side parallel to the existing fence, find the dimension of the field of largest area that can be enclosed for \$1000.

9. A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 ft, find the dimensions that will allow for the maximum amount of light to enter.

10. A page of a book is to have an area of 90 in^2, with 1 in. margins at the bottom and sides and a ½ in margin at the top. Find the dimensions of the page that will allow for the largest printed area.

11. A farmer has 3600 ft of fencing with which to enclose 3 side-by-side rectangular pens. Find the dimensions that will provide the greatest total area.

12. A builder intends to construct a storage shed having a volume of 900 ft^3, a flat roof, and a rectangular base whose width is three-fourths the length. The cost per square foot of the materials is \$4 for the floor, \$6 for the sides and \$3 for the roof. What dimensions will minimize the cost?

13. If a box with a square base and an open top is to have a volume of 4 ft^3, find the dimensions that require the least material.

14. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 ft, find the dimensions of the rectangle that will produce the largest area for the window.

1. 3 *2. 3* *3. $r = 2, h = 6$* *4. $x = 6, h = 3$* *5. $x = 12, y = 24$* *6. $x = 6, y = 4$, so the page is 9 by 6*