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Two airplanes leave the same airport at the same time. After completing their take-offs, the planes assume flight paths that form an angle of  $55^\circ$ . Their speeds are 450 mi/h and 600 mi/h. How far apart, to two significant digits, are the planes after 2 h?

The lengths of the three sides of a triangular lot are 40 m, 50 m, and 80 m. Find, to the nearest degree, the measure of the largest angle of the triangular lot.

The angle of elevation to the top of a mountain from a point  $P$  on the ground is  $24^\circ 10'$ . The angle of elevation from a point  $Q$  directly in line with  $P$  and 1,350 ft closer is  $61^\circ 40'$ . Find the height  $h$  of the mountain to three significant digits.

A ship is sailing due north. The captain observes that the bearing of a lighthouse is  $40^\circ$ . After sailing 60 km, the captain sees that the bearing of the lighthouse has become  $135^\circ$ . How far, to two significant digits, is the ship from the lighthouse now?

To determine the distance  $AB$  across a steep canyon, Megan walks 600 yd from  $B$  to another point,  $C$ . She then finds that  $m\angle ACB = 35^\circ$  and  $m\angle CBA = 106^\circ$ . Find  $AB$ .

The distance between Towns  $A$  and  $B$  is 56 mi. The angle formed by the road between Towns  $A$  and  $B$  and the road between Towns  $A$  and  $C$  measures  $46^\circ$ . The angle formed by  $\overline{AB}$  and  $\overline{BC}$  measures  $115^\circ$ . Find the distance between Town  $B$  and Town  $C$ .

A ship is steaming south. The navigator notices that the bearing of a lighthouse is  $120^\circ$ . After moving 8.0 mi/h for 2 h, he observes that the bearing of the lighthouse is  $25^\circ$ . Find his distance from the lighthouse at the time of the second sighting.

After two airplanes left the same airport at the same time, their flight paths formed an angle measuring  $125^\circ$ . The first flew at 550 mi/h and the second flew at 620 mi/h. How far apart were they after 3 h?

The sides of an isosceles triangle have lengths 18, 18, and 10. Find the measure of the smallest angle of the triangle.

A triangular lot has side lengths of 16 m, 26 m, and 38 m. Find the measure of the largest angle of the lot.

On a ship sailing north, a woman notices that a hotel on shore has a bearing of  $20^\circ$ . A little while later, after having sailed 40 km, she observes that the bearing of the hotel is now  $100^\circ$ . How far is the ship from the hotel now?

Bill determines that the angle of elevation to the top of a building measures  $40^\circ 30'$ . If he walks 102 ft closer to the building, the measure of the new angle of elevation will be  $50^\circ 20'$ . Find the height of the building to three significant digits.

A baseball diamond forms a square 90 ft on a side. The pitcher's mound is 60 ft from home plate. How far is it from the mound to third base?

The length of the radius of a circle is 10. Two radii,  $\overline{OA}$  and  $\overline{OB}$ , form an angle of measure  $109^\circ 30'$ . Find the length of chord  $\overline{AB}$ .

The diagonals of a parallelogram bisect each other. If their lengths are 8.0 and 10 and they intersect at an angle of  $20^\circ 40'$ , how long are the sides?

9a →

Ba →

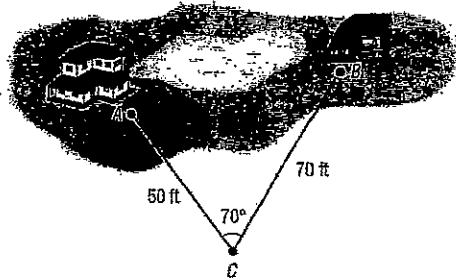
# Law of Sine/cosines

(2)

## TRIANGLES

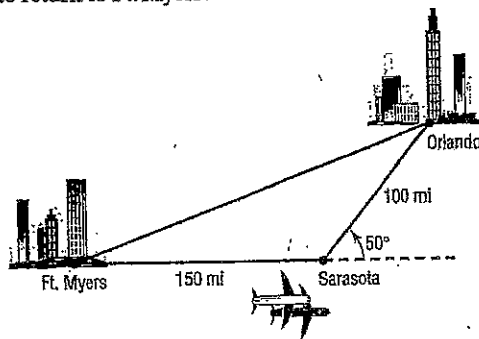
19. The diagonals of a parallelogram are 56 in. and 34 in. and intersect at an angle of  $120^\circ$ . Find the length of the shorter side.
20. The diagonals of a parallelogram are 14 m and 16 m and intersect at an angle of  $60^\circ$ . Find the length of the longer side.
21. Two planes leave an airport at the same time. Their speeds are 130 mph and 150 mph, and the angle between their courses is  $36^\circ$ . How far apart are they after 1.5 hours?
22. Two ships leave a harbor entrance at the same time. The first ship is traveling at a constant 18 mph, while the second is traveling at a constant 22 mph. If the angle between their courses is  $123^\circ$ , how far apart are they after two hours?
23. Two planes take off at the same time from an airport. The first plane is flying at 246 mph on a course of  $135.0^\circ$ . The second plane is flying in the direction  $175.0^\circ$  at 357 mph. Assuming there are no wind currents blowing, how far apart are they after 2 hours? What is the bearing of the second plane from the first at that time?
24. Two ships leave the harbor at the same time. One ship is traveling at 14 mph on a course with a bearing of  $S 13^\circ W$ , while the other is traveling at 12 mph on a course with a bearing of  $N 75^\circ E$ . How far apart are they after three hours?

25. **Surveying** Consult the figure. To find the distance from the house at  $A$  to the house at  $B$ , a surveyor measures the angle  $ACB$ , which is found to be  $70^\circ$ , and then walks off the distance to each house, 50 feet and 70 feet, respectively. How far apart are the houses?



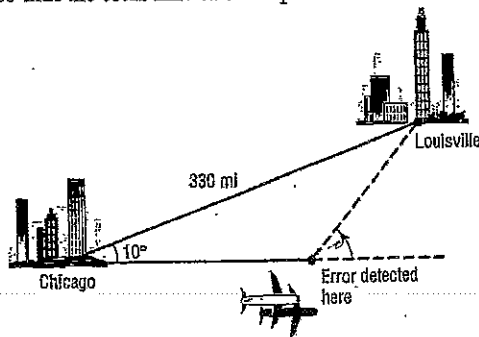
26. **Navigation** An airplane flies from Ft. Myers to Sarasota, a distance of 150 miles, and then turns through an angle of  $50^\circ$  and flies to Orlando, a distance of 100 miles (see the figure).

- (a) How far is it from Ft. Myers to Orlando?  
 (b) Through what angle should the pilot turn at Orlando to return to Ft. Myers?



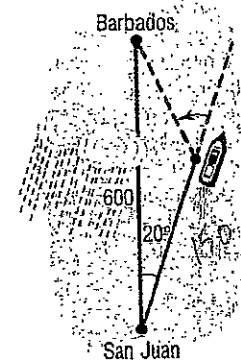
27. **Revising a Flight Plan** In attempting to fly from Chicago to Louisville, a distance of 330 miles, a pilot inadvertently took a course that was  $10^\circ$  in error, as indicated in the figure.

- (a) If the aircraft maintains an average speed of 220 miles per hour and if the error in direction is discovered after 15 minutes, through what angle should the pilot turn to head toward Louisville?  
 (b) What new average speed should the pilot maintain so that the total time of the trip is 90 minutes?



28. **Avoiding a Tropical Storm** A cruise ship maintains an average speed of 15 knots in going from San Juan, Puerto Rico, to Barbados, West Indies, a distance of 600 nautical miles. To avoid a tropical storm, the captain heads out of San Juan in a direction of  $20^\circ$  off a direct heading to Barbados. The captain maintains the 15-knot speed for 10 hours, after which time the path to Barbados becomes clear of storms.

- (a) Through what angle should the captain turn to head directly to Barbados?  
 (b) How long will it be before the ship reaches Barbados if the same 15-knot speed is maintained?



29. **Major League Baseball Field** A Major League baseball diamond is actually a square 90 feet on a side. The pitching rubber is located 60.5 feet from home plate on a line joining home plate and second base.

- (a) How far is it from the pitching rubber to first base?  
 (b) How far is it from the pitching rubber to second base?  
 (c) If a pitcher faces home plate, through what angle does he need to turn to face first base?

30. **Little League Baseball Field** According to Little League baseball official regulations, the diamond is a square 60 feet on a side. The pitching rubber is located 46 feet from home plate on a line joining home plate and second base.

- (a) How far is it from the pitching rubber to first base?  
 (b) How far is it from the pitching rubber to second base?  
 (c) If a pitcher faces home plate, through what angle does he need to turn to face first base?

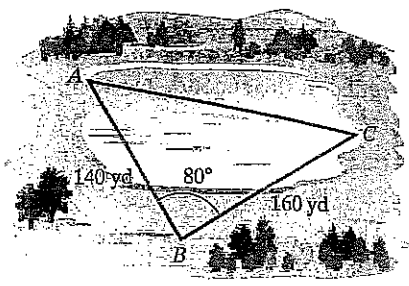
31. **Finding the Length of a Guy Wire** The height of a radio tower is 500 feet, and the ground on one side of the tower slopes upward at an angle of  $10^\circ$  (see the figure on p. 538).

- (a) How long should a guy wire be if it is to connect to the top of the tower and be secured at a point on the sloped side 100 feet from the base of the tower?  
 (b) How long should a second guy wire be if it is to connect to the middle of the tower and be secured at a point 100 feet from the base on the flat side?

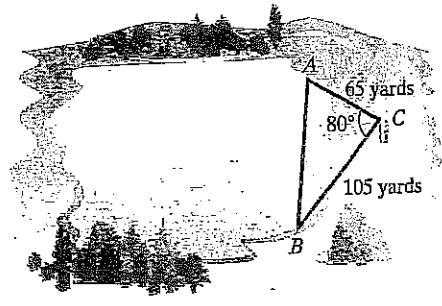
1. Two ships leave a harbor at the same time. One ship travels on a bearing of  $S12^\circ W$  at 14 miles per hour. The other travels on a bearing of  $N75^\circ E$  at 10 miles per hour. How far apart will the ships be after three hours? Round to the nearest tenth of a mile.

2. A plane leaves airport A and travels 580 miles to airport B on a bearing of  $N34^\circ E$ . The plane later leaves airport B and travels to airport C 400 miles away on a bearing of  $S7^\circ E$ . Find the distance from airport A to airport C to the nearest tenth of a mile.

3. Find the distance across the lake from A to C, to the nearest yard, using the measurements shown in the figure.



4. To find the distance across a protected cove at a lake, a surveyor makes the measurements shown in the figure. Use these measurements to find the distance from A to B to the nearest yard.



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5. You are on island A, on what bearing should you navigate to go to island C?

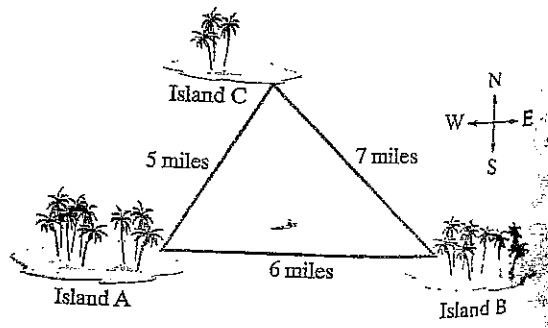
6. You are on island B, on what bearing should you navigate to go to island C?

7. You are on a fishing boat that leaves its pier and heads east. After traveling for 25 miles, there is a report warning of rough seas directly south. The captain turns the boat and follows a bearing of  $S40^\circ W$  for 13.5 miles.

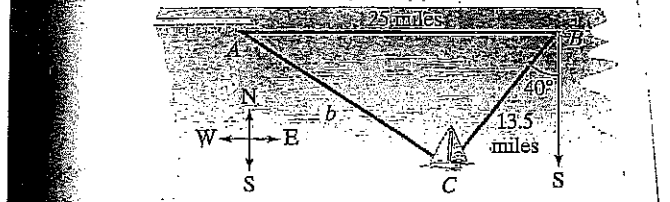
a. At this time, how far are you from the boat's pier? Round to the nearest tenth of a mile.

b. What bearing could the boat have originally taken to arrive at this spot?

The diagram shows three islands in Florida Bay. You rent a boat and plan to visit each of these remote islands. Use the diagram to solve Exercises 43-44.



8. You are on a fishing boat that leaves its pier and heads east. After traveling for 30 miles, there is a report warning of rough seas directly south. The captain turns the boat and follows a bearing of  $S45^\circ W$  for 12 miles.



a. At this time, how far are you from the boat's pier? Round to the nearest tenth of a mile.

b. What bearing could the boat have originally taken to arrive at this spot?

(9) A Major League baseball diamond has four bases forming a square whose sides measure 90 feet each. The pitcher's mound is 60.5 feet from home plate on a line joining home plate and second base. Find the distance from the pitcher's mound to first base. Round to the nearest tenth of a foot.

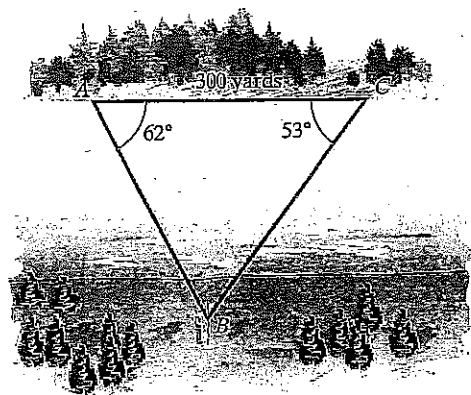
(10) A Little League baseball diamond has four bases forming a square whose sides measure 60 feet each. The pitcher's mound is 46 feet from home plate on a line joining home plate and second base. Find the distance from the pitcher's mound to third base. Round to the nearest tenth of a foot.

### Application Exercises

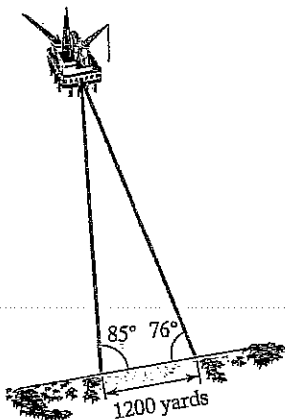
(11) Two fire-lookout stations are 10 miles apart, with station B 6 miles east of station A. Both stations spot a fire. The bearing of the fire from station A is  $N25^\circ E$  and the bearing of the fire from station B is  $N56^\circ W$ . How far, to the nearest tenth of a mile, is the fire from each lookout station?

(12) The Federal Communications Commission is attempting to locate an illegal radio station. It sets up two monitoring stations, A and B, with station B 40 miles east of station A. Station A measures the illegal signal from the radio station as coming from a direction of  $48^\circ$  east of north. Station B measures the signal as coming from a point  $34^\circ$  west of north. How far is the illegal radio station from monitoring stations A and B? Round to the nearest tenth of a mile.

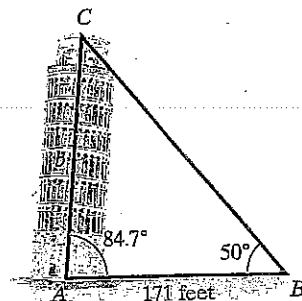
(14) A surveyor needs to determine the distance between two points that lie on opposite banks of a river. The figure shows that 300 yards are measured along one bank. The angles from each end of this line segment to a point on the opposite bank are  $62^\circ$  and  $53^\circ$ . Find the distance between A and B to the nearest tenth of a yard.



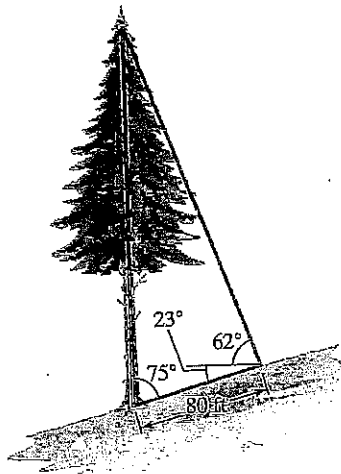
(13) The figure shows a 1200-yard-long sand beach and an oil platform in the ocean. The angle made with the platform from one end of the beach is  $85^\circ$  and from the other end is  $76^\circ$ . Find the distance of the oil platform, to the nearest tenth of a yard, from each end of the beach.



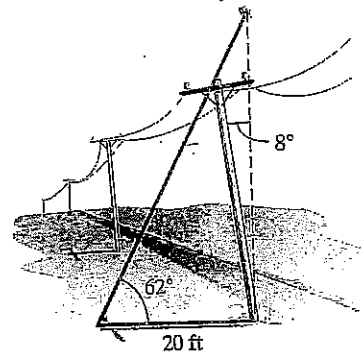
(15) The Leaning Tower of Pisa in Italy leans at an angle of about  $84.7^\circ$ . The figure shows that 171 feet from the base of the tower, the angle of elevation to the top is  $50^\circ$ . Find the distance, to the nearest tenth of a foot, from the base to the top of the tower.



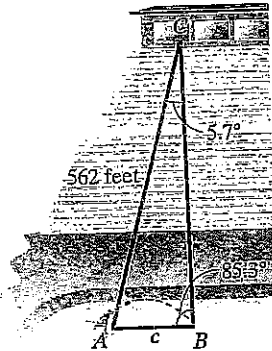
- 16) A pine tree growing on a hillside makes a  $75^\circ$  angle with the hill. From a point 80 feet up the hill, the angle of elevation to the top of the tree is  $62^\circ$  and the angle of depression to the bottom is  $23^\circ$ . Find, to the nearest tenth of a foot, the height of the tree.



- 19) When the angle of elevation of the sun is  $62^\circ$ , a telephone pole that is tilted at an angle of  $8^\circ$  directly away from the sun casts a shadow 20 feet long. Determine the length of the pole to the nearest tenth of a foot.



- 17) The figure shows a shot-put ring. The shot is tossed from  $A$  and lands at  $B$ . Using modern electronic equipment, the distance of the toss can be measured without the use of measuring tapes. When the shot lands at  $B$ , an electronic transmitter placed at  $B$  sends a signal to a device in the official's booth above the track. The device determines the angles at  $B$  and  $C$ . At a track meet, the distance from the official's booth to the shot-put ring is 562 feet. If  $B = 85.3^\circ$  and  $C = 5.7^\circ$ , determine the length of the toss to the nearest tenth of a foot.



- 20) A leaning wall is inclined  $6^\circ$  from the vertical. At a distance of 40 feet from the wall, the angle of elevation to the top is  $22^\circ$ . Find the height of the wall to the nearest tenth of a foot.

- 18) A pier forms an  $85^\circ$  angle with a straight shore. At a distance of 100 feet from the pier, the line of sight to the tip forms a  $37^\circ$  angle. Find the length of the pier to the nearest tenth of a foot.

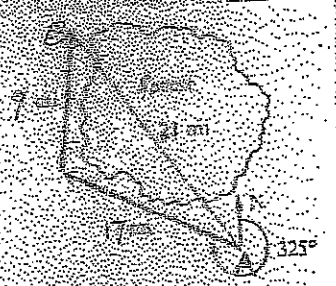
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Ex 1 A ship leaves port on a bearing of  $28^\circ$  and travels 9.2 miles. The ship then turns due east and travels 4.3 miles. How far is the ship from port? What is its bearing from port?

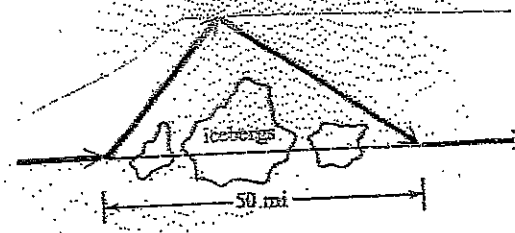
Ex 2

A plane with an airspeed of 192 mph is flying on a bearing of  $121^\circ$ . A north wind is blowing (from north to south) 15 mph. Find the groundspeed and the actual bearing of the plane.

- 13. A plane flies 650 mph on a bearing of  $175.3^\circ$ . A 25 mph wind, from a direction of  $266.6^\circ$ , blows against the plane. Find the resulting bearing of the plane.
- 14. A pilot wants to fly on a bearing of  $74.9^\circ$ . By flying due east, he finds that a 42 mph wind, blowing from the south, puts him on course. Find the airspeed and the groundspeed.
- 15. Starting at point A, a ship sails 18.5 km on a bearing of  $189^\circ$ , then turns and sails 47.8 km on a bearing of  $317^\circ$ . Find the distance of the ship from point A.
- 16. Two towns 21 mi apart are separated by a dense forest. (See the figure.) To travel from town A to town B, a person must go 17 mi on a bearing of  $325^\circ$ , then turn and continue for 9 mi to reach town B. Find the bearing of B from A.

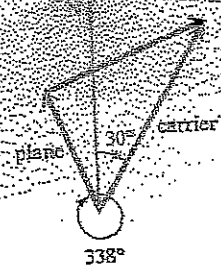


- 17. The airline route from San Francisco to Honolulu is on a bearing of  $233^\circ$ . A jet flying at 450 mph on that bearing flies into a wind blowing at 39 mph from a direction of  $114^\circ$ . Find the resulting bearing and groundspeed of the plane.
- 18. A pilot is flying at 168 mph. She wants her flight path to be on a bearing of  $57^\circ 40'$ . A wind is blowing from the south at 27.1 mph. Find the bearing the pilot should fly, and find the plane's groundspeed.
- 19. What bearing and airspeed are required for a plane to fly 400 mi due north in 2.5 hr if the wind is blowing from a direction of  $328^\circ$  at 11 mph?
- 20. A plane is headed due south with an airspeed of 192 mph. A wind from a direction of  $78^\circ$  is blowing at 23 mph. Find the groundspeed and resulting bearing of the plane.
- 21. An airplane is heading on a bearing of  $174^\circ$  at an airspeed of 240 km per hr. A 30 km per hr wind is blowing from a direction of  $245^\circ$ . Find the groundspeed and resulting bearing of the plane.
- 22. A ship sailing the North Atlantic has been warned to change course to avoid a group of icebergs. The captain turns and sails on a bearing of  $62^\circ$  for a while, then changes course again to a bearing of  $115^\circ$  until the ship reaches its original course. (See the figure.) How much farther did the ship have to travel to avoid the icebergs?



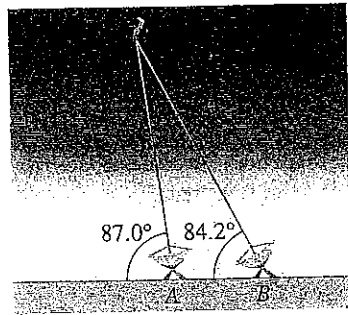
# 22

23. The aircraft carrier *Tallahassee* is traveling at sea on a steady course with a bearing of  $30^\circ$  at 32 mph. Patrol planes on the carrier have enough fuel for 2.6 hr of flight when traveling at a speed of 520 mph. One of the pilots takes off on a bearing of  $338^\circ$  and then turns and heads in a straight line, so as to be able to catch the carrier and land on the deck at the exact instant that his fuel runs out. If the pilot left at 2 p.m., at what time did he turn to head for the carrier?



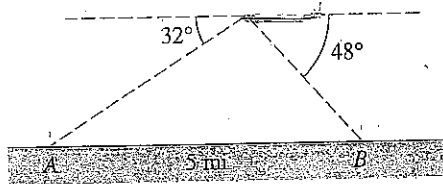
side of the two stations, the angles of elevation at  $A$  and  $B$  are measured to be  $87.0^\circ$  and  $84.2^\circ$ , respectively.

- How far is the satellite from station  $A$ ?
- How high is the satellite above the ground?

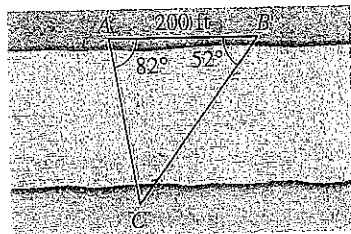


- Flight of a Plane** A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 5 mi apart, to be  $32^\circ$  and  $48^\circ$ , as shown in the figure.

  - Find the distance of the plane from point  $A$ .
  - Find the elevation of the plane.



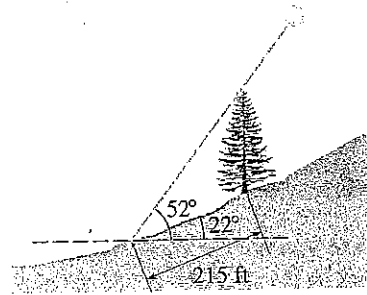
- Distance Across a River** To find the distance across a river, a surveyor chooses points  $A$  and  $B$ , which are 200 ft apart on one side of the river (see the figure). She then chooses a reference point  $C$  on the opposite side of the river and finds that  $\angle BAC \approx 82^\circ$  and  $\angle ABC \approx 52^\circ$ . Approximate the distance from  $A$  to  $C$ .



- Distance Across a Lake** Points  $A$  and  $B$  are separated by a lake. To find the distance between them, a surveyor locates a point  $C$  on land such that  $\angle CAB = 48.6^\circ$ . He also measures  $CA$  as 312 ft and  $CB$  as 527 ft. Find the distance between  $A$  and  $B$ .
- The Leaning Tower of Pisa** The bell tower of the cathedral in Pisa, Italy, leans  $5.6^\circ$  from the vertical. A tourist stands 105 m from its base, with the tower leaning directly toward her. She measures the angle of elevation to the top of the tower to be  $29.2^\circ$ . Find the length of the tower to the nearest meter.

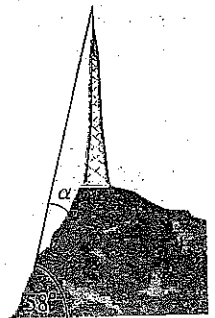
- Radio Antenna** A short-wave radio antenna is supported by two guy wires, 165 ft and 180 ft long. Each wire is attached to the top of the antenna and anchored to the ground, at two anchor points on opposite sides of the antenna. The shorter wire makes an angle of  $67^\circ$  with the ground. How far apart are the anchor points?

- Height of a Tree** A tree on a hillside casts a shadow 215 ft down the hill. If the angle of inclination of the hillside is  $22^\circ$  to the horizontal and the angle of elevation of the sun is  $52^\circ$ , find the height of the tree.

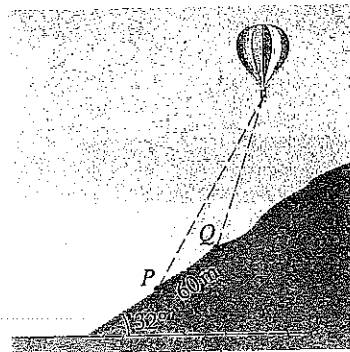


- Length of a Guy Wire**

A communications tower is located at the top of a steep hill, as shown. The angle of inclination of the hill is  $58^\circ$ . A guy wire is to be attached to the top of the tower and to the ground, 100 m downhill from the base of the tower. The angle  $\alpha$  in the figure is determined to be  $12^\circ$ . Find the length of cable required for the guy wire.



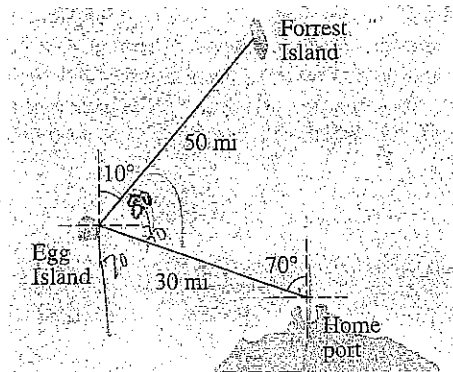
- Calculating a Distance** Observers at  $P$  and  $Q$  are located on the side of a hill that is inclined  $32^\circ$  to the horizontal, as shown. The observer at  $P$  determines the angle of elevation to a hot-air balloon to be  $62^\circ$ . At the same instant, the observer at  $Q$  measures the angle of elevation to the balloon to be  $71^\circ$ . If  $P$  is 60 m down the hill from  $Q$ , find the distance from  $Q$  to the balloon.





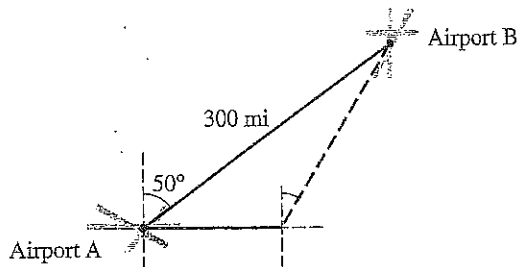
43. **Navigation** A fisherman leaves his home port and heads in the direction  $N 70^\circ W$ . He travels 30 mi and reaches Egg Island. The next day he sails  $N 10^\circ E$  for 50 mi, reaching Forrest Island.

- (a) Find the distance between the fisherman's home port and Forrest Island.  
 (b) Find the bearing from Forrest Island back to his home port.



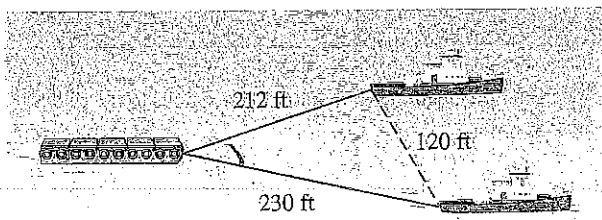
44. **Navigation** Airport B is 300 mi from airport A at a bearing  $N 50^\circ E$  (see the figure). A pilot wishing to fly from A to B mistakenly flies due east at 200 mi/h for 30 minutes, when he notices his error.

- (a) How far is the pilot from his destination at the time he notices the error?  
 (b) What bearing should he head his plane in order to arrive at airport B?

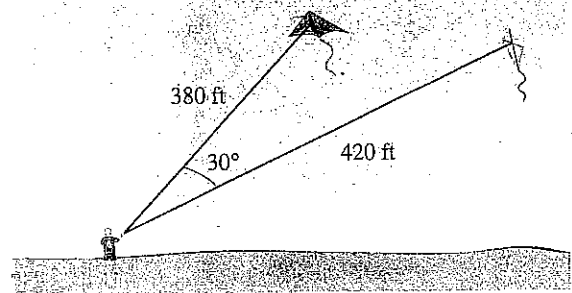


45. **Triangular Field** A triangular field has sides of lengths 22, 36, and 44 yd. Find the largest angle.

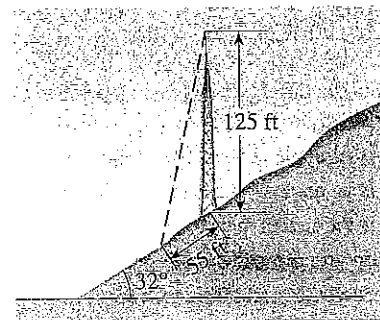
46. **Towing a Barge** Two tugboats that are 120 ft apart pull a barge, as shown. If the length of one cable is 212 ft and the length of the other is 230 ft, find the angle formed by the two cables.



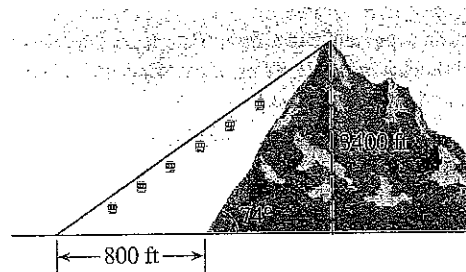
47. **Flying Kites** A boy is flying two kites at the same time. He has 380 ft of line out to one kite and 420 ft to the other. He estimates the angle between the two lines to be  $30^\circ$ . Approximate the distance between the kites.



48. **Securing a Tower** A 125-ft tower is located on the side of a mountain that is inclined  $32^\circ$  to the horizontal. A guy wire is to be attached to the top of the tower and anchored at a point 55 ft downhill from the base of the tower. Find the shortest length of wire needed.



49. **Cable Car** A steep mountain is inclined  $74^\circ$  to the horizontal and rises 3400 ft above the surrounding plain. A cable car is to be installed from a point 800 ft from the base to the top of the mountain, as shown. Find the shortest length of cable needed.



50. **CN Tower** The CN Tower in Toronto, Canada, is the tallest free-standing structure in the world. A woman on the observation deck, 1150 ft above the ground, wants to determine the distance between two landmarks on the ground below. She observes that the angle formed by the lines of sight to these two landmarks is  $43^\circ$ . She also observes that the angle between the vertical and the line of sight to one of the