we should begin by multiplying both sides by $\sin x(1 + \cos x)$, the least common denominator. _____

## EXERCISE SET 6.1

### Practice Exercises

*In Exercises 1–60, verify each identity.*

**1.** $\sin x \sec x = \tan x$

**2.** $\cos x \csc x = \cot x$

**3.** $\tan(-x)\cos x = -\sin x$

**4.** $\cot(-x)\sin x = -\cos x$

**5.** $\tan x \csc x \cos x = 1$

**6.** $\cot x \sec x \sin x = 1$

**7.** $\sec x - \sec x \sin^2 x = \cos x$

**8.** $\csc x - \csc x \cos^2 x = \sin x$

**9.** $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

**10.** $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

**11.** $\csc\theta - \sin\theta = \cot\theta\cos\theta$

**12.** $\tan\theta + \cot\theta = \sec\theta\csc\theta$

**13.** $\dfrac{\tan\theta\cot\theta}{\csc\theta} = \sin\theta$

**14.** $\dfrac{\cos\theta\sec\theta}{\cot\theta} = \tan\theta$

**15.** $\sin^2\theta(1 + \cot^2\theta) = 1$

**16.** $\cos^2\theta(1 + \tan^2\theta) = 1$

**17.** $\sin t \tan t = \dfrac{1 - \cos^2 t}{\cos t}$

**18.** $\cos t \cot t = \dfrac{1 - \sin^2 t}{\sin t}$

**19.** $\dfrac{\csc^2 t}{\cot t} = \csc t \sec t$

**20.** $\dfrac{\sec^2 t}{\tan t} = \sec t \csc t$

**21.** $\dfrac{\tan^2 t}{\sec t} = \sec t - \cos t$

**22.** $\dfrac{\cot^2 t}{\csc t} = \csc t - \sin t$

**23.** $\dfrac{1 - \cos\theta}{\sin\theta} = \csc\theta - \cot\theta$

**24.** $\dfrac{1 - \sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

**25.** $\dfrac{\sin t}{\csc t} + \dfrac{\cos t}{\sec t} = 1$

**26.** $\dfrac{\sin t}{\tan t} + \dfrac{\cos t}{\cot t} = \sin t + \cos t$

**27.** $\tan t + \dfrac{\cos t}{1 + \sin t} = \sec t$

**28.** $\cot t + \dfrac{\sin t}{1 + \cos t} = \csc t$

**29.** $1 - \dfrac{\sin^2 x}{1 + \cos x} = \cos x$

**30.** $1 - \dfrac{\cos^2 x}{1 + \sin x} = \sin x$

**31.** $\dfrac{\cos x}{1 - \sin x} + \dfrac{1 - \sin x}{\cos x} = 2\sec x$

**32.** $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x - 1}{\sin x} = 0$

**33.** $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

**34.** $\csc^2 x \sec x = \sec x + \csc x \cot x$

**35.** $\dfrac{\sec x - \csc x}{\sec x + \csc x} = \dfrac{\tan x - 1}{\tan x + 1}$

**36.** $\dfrac{\csc x - \sec x}{\csc x + \sec x} = \dfrac{\cot x - 1}{\cot x + 1}$

**37.** $\dfrac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \sin x - \cos x$

**38.** $\dfrac{\tan^2 x - \cot^2 x}{\tan x + \cot x} = \tan x - \cot x$

**39.** $\tan^2 2x + \sin^2 2x + \cos^2 2x = \sec^2 2x$

**40.** $\cot^2 2x + \cos^2 2x + \sin^2 2x = \csc^2 2x$

**41.** $\dfrac{\tan 2\theta + \cot 2\theta}{\csc 2\theta} = \sec 2\theta$

**42.** $\dfrac{\tan 2\theta + \cot 2\theta}{\sec 2\theta} = \csc 2\theta$

**43.** $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

**44.** $\dfrac{\cot x + \cot y}{1 - \cot x \cot y} = \dfrac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y}$

**45.** $(\sec x - \tan x)^2 = \dfrac{1 - \sin x}{1 + \sin x}$

**46.** $(\csc x - \cot x)^2 = \dfrac{1 - \cos x}{1 + \cos x}$

**47.** $\dfrac{\sec t + 1}{\tan t} = \dfrac{\tan t}{\sec t - 1}$

**48.** $\dfrac{\csc t - 1}{\cot t} = \dfrac{\cot t}{\csc t + 1}$

**49.** $\dfrac{1 + \cos t}{1 - \cos t} = (\csc t + \cot t)^2$

**50.** $\dfrac{\cos^2 t + 4\cos t + 4}{\cos t + 2} = \dfrac{2\sec t + 1}{\sec t}$

**51.** $\cos^4 t - \sin^4 t = 1 - 2\sin^2 t$

**57.** $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = 2$

**58.** $(3\cos\theta - 4\sin\theta)^2 + (4\cos\theta + 3\sin\theta)^2 = 25$

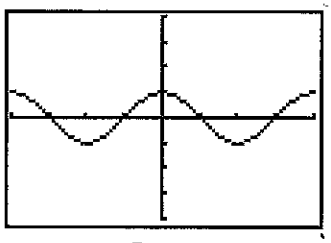
**59.** $\dfrac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$

**60.** $\dfrac{\sin x + \cos x}{\sin x} - \dfrac{\cos x - \sin x}{\cos x} = \sec x \csc x$

### Practice Plus

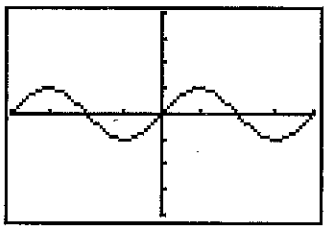
*In Exercises 61–66, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture.*

**61.** $\dfrac{(\sec x + \tan x)(\sec x - \tan x)}{\sec x} = ?$



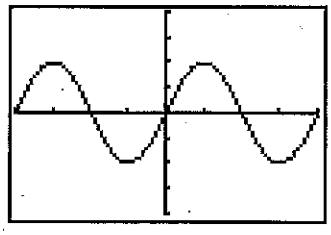
$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

**62.** $\dfrac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x} = ?$



$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

**63.** $\dfrac{\cos x + \cot x \sin x}{\cot x} = ?$



$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

**64.** $\dfrac{\cos x \tan x - \tan x + 2\cos x - 2}{\tan x + 2} = ?$