

10. Simplify each trigonometric expression by following the indicated direction.

Write in terms of sine and cosine functions:

$$\tan \theta \cdot \csc \theta.$$

$$\cot \theta \cdot \sec \theta.$$

13. Multiply $\frac{\cos \theta}{1 - \sin \theta}$ by $\frac{1 + \sin \theta}{1 + \sin \theta}$.

14. Multiply $\frac{\sin \theta}{1 + \cos \theta}$ by $\frac{1 - \cos \theta}{1 - \cos \theta}$.

15. Rewrite over a common denominator:

$$\frac{\sin \theta + \cos \theta}{\cos \theta} + \frac{\cos \theta - \sin \theta}{\sin \theta}$$

16. Rewrite over a common denominator:

$$\frac{1}{1 - \cos v} + \frac{1}{1 + \cos v}$$

17. Multiply and simplify: $\frac{(\sin \theta + \cos \theta)(\sin \theta + \cos \theta) - 1}{\sin \theta \cos \theta}$

18. Multiply and simplify: $\frac{(\tan \theta + 1)(\tan \theta + 1) - \sec^2 \theta}{\tan \theta}$

19. Factor and simplify: $\frac{3 \sin^2 \theta + 4 \sin \theta + 1}{\sin^2 \theta + 2 \sin \theta + 1}$

20. Factor and simplify: $\frac{\cos^2 \theta - 1}{\cos^2 \theta - \cos \theta}$

In Problems 21–100, establish each identity.

21. $\csc \theta \cdot \cos \theta = \cot \theta$

22. $\sec \theta \cdot \sin \theta = \tan \theta$

23. $1 + \tan^2(-\theta) = \sec^2 \theta$

24. $1 + \cot^2(-\theta) = \csc^2 \theta$

25. $\cos \theta(\tan \theta + \cot \theta) = \csc \theta$

26. $\sin \theta(\cot \theta + \tan \theta) = \sec \theta$

27. $\tan u \cot u - \cos^2 u = \sin^2 u$

28. $\sin u \csc u - \cos^2 u = \sin^2 u$

29. $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$

30. $(\csc \theta - 1)(\csc \theta + 1) = \cot^2 \theta$

31. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

32. $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$

33. $\cos^2 \theta(1 + \tan^2 \theta) = 1$

34. $(1 - \cos^2 \theta)(1 + \cot^2 \theta) = 1$

35. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

36. $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta = 1$

37. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

38. $\csc^4 \theta - \csc^2 \theta = \cot^4 \theta + \cot^2 \theta$

39. $\sec u - \tan u = \frac{\cos u}{1 + \sin u}$

40. $\csc u - \cot u = \frac{\sin u}{1 + \cos u}$

41. $3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta$

42. $9 \sec^2 \theta - 5 \tan^2 \theta = 5 + 4 \sec^2 \theta$

43. $1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$

44. $1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$

45. $\frac{1 + \tan v}{1 - \tan v} = \frac{\cot v + 1}{\cot v - 1}$

46. $\frac{\csc v - 1}{\csc v + 1} = \frac{1 - \sin v}{1 + \sin v}$

47. $\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$

48. $\frac{\csc \theta - 1}{\cot \theta} = \frac{\cot \theta}{\csc \theta + 1}$

49. $\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{\csc \theta + 1}{\csc \theta - 1}$

50. $\frac{\cos \theta + 1}{\cos \theta - 1} = \frac{1 + \sec \theta}{1 - \sec \theta}$

51. $\frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} = 2 \sec v$

52. $\frac{\cos v}{1 + \sin v} + \frac{1 + \sin v}{\cos v} = 2 \sec v$

53. $\frac{\sin \theta}{\sin \theta - \cos \theta} = \frac{1}{1 - \cot \theta}$

54. $1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta$

55. $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$

56. $\frac{1 - \cos \theta}{1 + \cos \theta} = (\csc \theta - \cot \theta)^2$

57. $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$

58. $\frac{\cot \theta}{1 - \tan \theta} + \frac{\tan \theta}{1 - \cot \theta} = 1 + \tan \theta + \cot \theta$

59. $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$

60. $\frac{\sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{\tan \theta}{1 - \tan^2 \theta}$

61. $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \tan \theta + \sec \theta$

62. $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\sin \theta + 1}{\cos \theta}$

63. $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \sin^2 \theta - \cos^2 \theta$

64. $\frac{\sec \theta - \cos \theta}{\sec \theta + \cos \theta} = \frac{\sin^2 \theta}{1 + \cos^2 \theta}$

65. $\frac{\tan u - \cot u}{\tan u + \cot u} + 1 = 2 \sin^2 u$

66. $\frac{\tan u - \cot u}{\tan u + \cot u} + 2 \cos^2 u = 1$

67. $\frac{\sec \theta + \tan \theta}{\cot \theta + \cos \theta} = \tan \theta \sec \theta$

68. $\frac{\sec \theta}{1 + \sec \theta} = \frac{1 - \cos \theta}{\sin^2 \theta}$

69. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} + 1 = 2 \cos^2 \theta$

70. $\frac{1 - \cot^2 \theta}{1 + \cot^2 \theta} + 2 \cos^2 \theta = 1$

71. $\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta} = \sin \theta - \cos \theta$

72. $\frac{\sin^2 \theta - \tan \theta}{\cos^2 \theta - \cot \theta} = \tan^2 \theta$

73. $\sec \theta - \cos \theta = \sin \theta \tan \theta$

74. $\tan \theta + \cot \theta = \sec \theta \csc \theta$

75. $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \sec^2 \theta$

76. $\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta} = 4 \tan \theta \sec \theta$

77. $\frac{\sec \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos^3 \theta}$ 78. $\frac{1 + \sin \theta}{1 - \sin \theta} = (\sec \theta + \tan \theta)^2$ 79. $\frac{(\sec v - \tan v)^2 + 1}{\csc v (\sec v - \tan v)} = 2 \tan v$
80. $\frac{\sec^2 v - \tan^2 v + \tan v}{\sec v} = \sin v + \cos v$ 81. $\frac{\sin \theta + \cos \theta}{\cos \theta} - \frac{\sin \theta - \cos \theta}{\sin \theta} = \sec \theta \csc \theta$
82. $\frac{\sin \theta + \cos \theta}{\sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta} = \sec \theta \csc \theta$ 83. $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = 1 - \sin \theta \cos \theta$
84. $\frac{\sin^3 \theta + \cos^3 \theta}{1 - 2 \cos^2 \theta} = \frac{\sec \theta - \sin \theta}{\tan \theta - 1}$ 85. $\frac{\cos^2 \theta - \sin^2 \theta}{1 - \tan^2 \theta} = \cos^2 \theta$ 86. $\frac{\cos \theta + \sin \theta - \sin^3 \theta}{\sin \theta} = \cot \theta + \cos^2 \theta$
87. $\frac{(2 \cos^2 \theta - 1)^2}{\cos^4 \theta - \sin^4 \theta} = 1 - 2 \sin^2 \theta$ 88. $\frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} = \tan \theta - \cot \theta$ 89. $\frac{1 + \sin \theta + \cos \theta}{1 + \sin \theta - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$
90. $\frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} = \sec \theta + \tan \theta$ 91. $(a \sin \theta + b \cos \theta)^2 + (a \cos \theta - b \sin \theta)^2 = a^2 + b^2$
92. $(2a \sin \theta \cos \theta)^2 + a^2(\cos^2 \theta - \sin^2 \theta)^2 = a^2$ 93. $\frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \tan \alpha \tan \beta$
94. $(\tan \alpha + \tan \beta)(1 - \cot \alpha \cot \beta) + (\cot \alpha + \cot \beta)(1 - \tan \alpha \tan \beta) = 0$
95. $(\sin \alpha + \cos \beta)^2 + (\cos \beta + \sin \alpha)(\cos \beta - \sin \alpha) = 2 \cos \beta(\sin \alpha + \cos \beta)$
96. $(\sin \alpha - \cos \beta)^2 + (\cos \beta + \sin \alpha)(\cos \beta - \sin \alpha) = -2 \cos \beta(\sin \alpha - \cos \beta)$
97. $\ln |\sec \theta| = -\ln |\cos \theta|$ 98. $\ln |\tan \theta| = \ln |\sin \theta| - \ln |\cos \theta|$
99. $\ln |1 + \cos \theta| + \ln |1 - \cos \theta| = 2 \ln |\sin \theta|$ 100. $\ln |\sec \theta + \tan \theta| + \ln |\sec \theta - \tan \theta| = 0$
- In Problems 101–104, show that the functions f and g are identically equal.
101. $f(x) = \sin x \cdot \tan x$ $g(x) = \sec x - \cos x$ 102. $f(x) = \cos x \cdot \cot x$ $g(x) = \csc x - \sin x$
103. $f(\theta) = \frac{1 - \sin \theta}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta}$ $g(\theta) = 0$ 104. $f(\theta) = \tan \theta + \sec \theta$ $g(\theta) = \frac{\cos \theta}{1 - \sin \theta}$

Applications and Extensions

105. **Searchlights** A searchlight at the grand opening of a new car dealership casts a spot of light on a wall located 75 meters from the searchlight. The acceleration \ddot{r} of the spot of light is found to be $\ddot{r} = 1200 \sec \theta (2 \sec^2 \theta - 1)$. Show that this is equivalent to $\ddot{r} = 1200 \left(\frac{1 + \sin^2 \theta}{\cos^3 \theta} \right)$.
- Source: Adapted from Hibbeler, *Engineering Mechanics: Dynamics*, 13th ed., Pearson © 2013.
106. **Optical Measurement** Optical methods of measurement often rely on the interference of two light waves. If two light waves, identical except for a phase lag, are mixed together, the resulting intensity, or irradiance, is given by
- $$I_i = 4A^2 \frac{(\csc \theta - 1)(\sec \theta + \tan \theta)}{\csc \theta \sec \theta}$$
- Show that this is equivalent to $I_i = (2A \cos \theta)^2$.
- Source: *Experimental Techniques*, July/August 2002

Explaining Concepts: Discussion and Writing

107. Write a few paragraphs outlining your strategy for establishing identities.
108. Write down the three Pythagorean Identities.
109. Why do you think it is usually preferable to start with the side containing the more complicated expression when establishing an identity?
110. Make up an identity that is not a basic identity.

Retain Your Knowledge

Problems 111–114 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

111. Determine whether $f(x) = -3x^2 + 120x + 50$ has a maximum or a minimum value, and then find the value.
112. Given $f(x) = \frac{x+1}{x-2}$ and $g(x) = 3x - 4$, find $f \circ g$.
113. Find the exact values of the six trigonometric functions of an angle θ in standard position if $(-12, 5)$ is a point on its terminal side.
114. Find the average rate of change of $f(x) = \cos x$ from 0 to $\frac{\pi}{2}$.

'Are You Prepared?' Answers

1. True 2. True