An even longer strategy to verify $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$, but one that works, is to replace each of the two occurrences of $\csc^2 x$ on the left side by $\frac{1}{\sin^2 x}$. This may be the approach that you first consider, particularly if you become accustomed to rewriting the more complicated side in terms of sines and cosines. The selection of an appropriate fundamental identity to solve the puzzle most efficiently is learned through lots of practice.

The more identities you prove, the more confident and efficient you will become. Although practice is the only way to learn how to verify identities, there are some guidelines developed throughout the section that should help you get started.

Guidelines for Verifying Trigonometric Identities

- Work with each side of the equation independently of the other side. Start with the more complicated side and transform it in a step-by-step fashion until it looks exactly like the other side.
- Analyze the identity and look for opportunities to apply the fundamental identities.
- Try using one or more of the following techniques:
  1. Rewrite the more complicated side in terms of sines and cosines.
  2. Factor out the greatest common factor.
  3. Separate a single-term quotient into two terms:
  $$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}.$$
  4. Combine fractional expressions using the least common denominator.
  5. Multiply the numerator and the denominator by a binomial factor that appears on the other side of the identity.
- Don't be afraid to stop and start over again if you are not getting anywhere. Creative puzzle solvers know that strategies leading to dead ends often provide good problem-solving ideas.

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. To verify an identity, start with the more __complicated__ side and transform it in a step-by-step fashion until it is identical to the __other__ side.
2. It is sometimes helpful to verify an identity by rewriting one of the sides in terms of __sines__ and __cosines__, and then simplifying the result.
3. True or false: To verify the identity
$$\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x},$$
we should begin by multiplying both sides by $\sin x(1 + \cos x)$, the least common denominator. __false__
4. $\frac{1}{\csc x - 1} - \frac{1}{\csc x + 1}$ can be simplified using __$(\csc x - 1)(\csc x + 1)$__ as the least common denominator.
5. You can use your graphing calculator to provide evidence of an identity. Graph the left side and the right side separately, and see if the two graphs are __identical/the same__.

EXERCISE SET 5.1

Practice Exercises

In Exercises 1–60, verify each identity. 1–60. Proofs may vary.

1. $\sin x \sec x = \tan x$
2. $\cos x \csc x = \cot x$
3. $\tan(-x)\cos x = -\sin x$
4. $\cot(-x) \sin x = -\cos x$
5. $\tan x \csc x \cos x = 1$
6. $\cot x \sec x \sin x = 1$
7. $\sec x - \sec x \sin^2 x = \cos x$
8. $\csc x - \csc x \cos^2 x = \sin x$
9. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$
10. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$
11. $\csc\theta - \sin\theta = \cot\theta\cos\theta$
12. $\tan\theta + \cot\theta = \sec\theta\csc\theta$
13. $\frac{\tan\theta\cot\theta}{\csc\theta} = \sin\theta$
14. $\frac{\cos\theta\sec\theta}{\cot\theta} = \tan\theta$
15. $\sin^2\theta(1 + \cot^2\theta) = 1$
16. $\cos^2\theta(1 + \tan^2\theta) = 1$
17. $\sin t \tan t = \frac{1 - \cos^2 t}{\cos t}$
18. $\cos t \cot t = \frac{1 - \sin^2 t}{\sin t}$
19. $\frac{\csc^2 t}{\cot t} = \csc t \sec t$
20. $\frac{\sec^2 t}{\tan t} = \sec t \csc t$
21. $\frac{\tan^2 t}{\sec t} = \sec t - \cos t$
22. $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$

23. $\dfrac{1-\cos\theta}{\sin\theta}=\csc\theta-\cot\theta$ **24.** $\dfrac{1-\sin\theta}{\cos\theta}=\sec\theta-\tan\theta$

25. $\dfrac{\sin t}{\csc t}+\dfrac{\cos t}{\sec t}=1$

26. $\dfrac{\sin t}{\tan t}+\dfrac{\cos t}{\cot t}=\sin t+\cos t$

27. $\tan t+\dfrac{\cos t}{1+\sin t}=\sec t$ **28.** $\cot t+\dfrac{\sin t}{1+\cos t}=\csc t$

29. $1-\dfrac{\sin^2 x}{1+\cos x}=\cos x$ **30.** $1-\dfrac{\cos^2 x}{1+\sin x}=\sin x$

31. $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=2\sec x$

32. $\dfrac{\sin x}{\cos x+1}+\dfrac{\cos x-1}{\sin x}=0$

33. $\sec^2 x\csc^2 x=\sec^2 x+\csc^2 x$

34. $\csc^2 x\sec x=\sec x+\csc x\cot x$

35. $\dfrac{\sec x-\csc x}{\sec x+\csc x}=\dfrac{\tan x-1}{\tan x+1}$ **36.** $\dfrac{\csc x-\sec x}{\csc x+\sec x}=\dfrac{\cot x-1}{\cot x+1}$

37. $\dfrac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\sin x-\cos x$

38. $\dfrac{\tan^2 x-\cot^2 x}{\tan x+\cot x}=\tan x-\cot x$

39. $\tan^2 2x+\sin^2 2x+\cos^2 2x=\sec^2 2x$

40. $\cot^2 2x+\cos^2 2x+\sin^2 2x=\csc^2 2x$

41. $\dfrac{\tan 2\theta+\cot 2\theta}{\csc 2\theta}=\sec 2\theta$ **42.** $\dfrac{\tan 2\theta+\cot 2\theta}{\sec 2\theta}=\csc 2\theta$

43. $\dfrac{\tan x+\tan y}{1-\tan x\tan y}=\dfrac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$

44. $\dfrac{\cot x+\cot y}{1-\cot x\cot y}=\dfrac{\cos x\sin y+\sin x\cos y}{\sin x\sin y-\cos x\cos y}$

45. $(\sec x-\tan x)^2=\dfrac{1-\sin x}{1+\sin x}$

46. $(\csc x-\cot x)^2=\dfrac{1-\cos x}{1+\cos x}$

47. $\dfrac{\sec t+1}{\tan t}=\dfrac{\tan t}{\sec t-1}$ **48.** $\dfrac{\csc t-1}{\cot t}=\dfrac{\cot t}{\csc t+1}$

49. $\dfrac{1+\cos t}{1-\cos t}=(\csc t+\cot t)^2$

50. $\dfrac{\cos^2 t+4\cos t+4}{\cos t+2}=\dfrac{2\sec t+1}{\sec t}$

51. $\cos^4 t-\sin^4 t=1-2\sin^2 t$

52. $\sin^4 t-\cos^4 t=1-2\cos^2 t$

53. $\dfrac{\sin\theta-\cos\theta}{\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta}=2-\sec\theta\csc\theta$

54. $\dfrac{\sin\theta}{1-\cot\theta}-\dfrac{\cos\theta}{\tan\theta-1}=\sin\theta+\cos\theta$

55. $(\tan^2\theta+1)(\cos^2\theta+1)=\tan^2\theta+2$

56. $(\cot^2\theta+1)(\sin^2\theta+1)=\cot^2\theta+2$

57. $(\cos\theta-\sin\theta)^2+(\cos\theta+\sin\theta)^2=2$

58. $(3\cos\theta-4\sin\theta)^2+(4\cos\theta+3\sin\theta)^2=25$

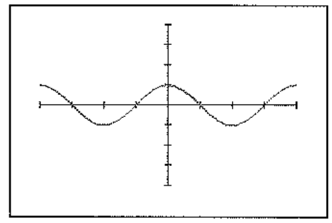
59. $\dfrac{\cos^2 x-\sin^2 x}{1-\tan^2 x}=\cos^2 x$

60. $\dfrac{\sin x+\cos x}{\sin x}-\dfrac{\cos x-\sin x}{\cos x}=\sec x\csc x$

Practice Plus

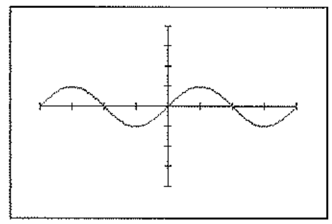
In Exercises 61–66, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture.

61. $\dfrac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x}=\,?$ cos x; Proofs may vary.



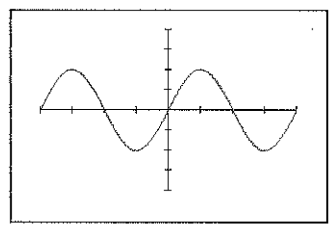
$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

62. $\dfrac{\sec^2 x\csc x}{\sec^2 x+\csc^2 x}=\,?$ sin x; Proofs may vary.



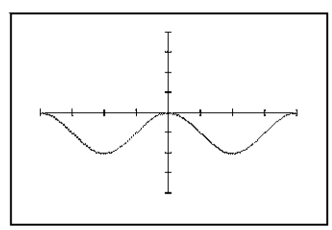
$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

63. $\dfrac{\cos x+\cot x\sin x}{\cot x}=\,?$ 2 sin x; Proofs may vary.



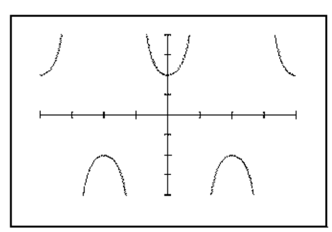
$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

64. $\dfrac{\cos x\tan x-\tan x+2\cos x-2}{\tan x+2}=\,?$ cos x − 1; Proofs may vary.



$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$

65. $\dfrac{1}{\sec x+\tan x}+\dfrac{1}{\sec x-\tan x}=\,?$ 2 sec x; Proofs may vary.



$[-2\pi, 2\pi, \frac{\pi}{2}]$ by $[-4, 4, 1]$