T F **5.** The present value of \$1000 to be received after 2 years at 10% per annum compounded continuously is approximately \$1205.

T F **6.** If $y = \log_a x$, then $y = a^x$.

T F **7.** The graph of every logarithmic function $f(x) = \log_a x$, $a > 0, a \neq 1$, will contain the points $(1, 0)$ and $(a, 1)$.

T F **8.** $a^{\log_M a} = M$, where $a > 0, a \neq 1, M > 0$.

T F **9.** $\log_a(M + N) = \log_a M + \log_a N$, where $a > 0, a \neq 1, M > 0, N > 0$.

T F **10.** $\log_a M - \log_a N = \log_a(M/N)$, where $a > 0, a \neq 1, M > 0, N > 0$.

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–6, the function f is one-to-one. Find the inverse of each function and check your answer. Find the domain and range of f and f^{-1}. Use a graphing utility to simultaneously graph f, f^{-1}, and $y = x$ on the same square screen.

1. $f(x) = \dfrac{2x + 3}{5x - 2}$ **2.** $f(x) = \dfrac{2 - x}{3 + x}$ **3.** $f(x) = \dfrac{1}{x - 1}$

4. $f(x) = \sqrt{x - 2}$ **5.** $f(x) = \dfrac{3}{x^{1/3}}$ **6.** $f(x) = x^{1/3} + 1$

In Problems 7–12, evaluate each expression. Do not use a graphing utility.

7. $\log_2(\frac{1}{8})$ **8.** $\log_3 81$ **9.** $\ln e^{\sqrt{2}}$

10. $e^{\ln 0.1}$ **11.** $2^{\log_2 0.4}$ **12.** $\log_2 2^{\sqrt{3}}$

In Problems 13–18, write each expression as the sum and/or difference of logarithms. Express powers as factors.

13. $\log_3\left(\dfrac{uv^2}{w}\right)$ **14.** $\log_2(a^2\sqrt{b})^4$ **15.** $\log(x^2\sqrt{x^3 + 1})$

16. $\log_5\left(\dfrac{x^2 + 2x + 1}{x^2}\right)$ **17.** $\ln\left(\dfrac{x\sqrt[3]{x^2 + 1}}{x - 3}\right)$ **18.** $\ln\left(\dfrac{2x + 3}{x^2 - 3x + 2}\right)^2$

In Problems 19–24, write each expression as a single logarithm.

19. $3\log_4 x^2 + \dfrac{1}{2}\log_4 \sqrt{x}$ **20.** $-2\log_3\left(\dfrac{1}{x}\right) + \dfrac{1}{3}\log_3 \sqrt{x}$

21. $\ln\left(\dfrac{x - 1}{x}\right) + \ln\left(\dfrac{x}{x + 1}\right) - \ln(x^2 - 1)$ **22.** $\log(x^2 - 9) - \log(x^2 + 7x + 12)$

23. $2\log 2 + 3\log x - \dfrac{1}{2}[\log(x + 3) + \log(x - 2)]$ **24.** $\dfrac{1}{2}\ln(x^2 + 1) - 4\ln\dfrac{1}{2} - \dfrac{1}{2}[\ln(x - 4) + \ln x]$

In Problems 25 and 26, use the Change-of-Base Formula and a calculator to evaluate each logarithm. Round your answer to three decimal places.

25. $\log_4 19$ **26.** $\log_2 21$

In Problems 27–32, find y as a function of x. The constant C is a positive number.

27. $\ln y = 2x^2 + \ln C$ **28.** $\ln(y - 3) = \ln 2x^2 + \ln C$

29. $\ln(y - 3) + \ln(y + 3) = x + C$ **30.** $\ln(y - 1) + \ln(y + 1) = -x + C$

31. $e^{y+C} = x^2 + 4$ **32.** $e^{3y-C} = (x + 4)^2$

In Problems 33–42, use transformations to graph each function. Determine the domain, range, and any asymptotes. Verify your results using a graphing utility.

33. $f(x) = 2^{x-3}$ **34.** $f(x) = -2^x + 3$ **35.** $f(x) = \dfrac{1}{2}(3^{-x})$ **36.** $f(x) = 1 + 3^{2x}$

37. $f(x) = 1 - e^x$ **38.** $f(x) = 3 + \ln x$ **39.** $f(x) = 3e^x$

40. $f(x) = \frac{1}{2}\ln x$ **41.** $f(x) = 3 - e^{-x}$ **42.** $f(x) = 4 - \ln(-x)$

In Problems 43–62, solve each equation. Verify your result using a graphing utility.

43. $4^{1-2x} = 2$ **44.** $8^{6+3x} = 4$ **45.** $3^{x^2+x} = \sqrt{3}$

46. $4^{x-x^2} = \frac{1}{2}$ **47.** $\log_x 64 = -3$ **48.** $\log_{\sqrt{2}} x = -6$

49. $5^x = 3^{x+2}$ **50.** $5^{x+2} = 7^{x-2}$ **51.** $9^{2x} = 27^{3x-4}$

52. $25^{2x} = 5^{x^2-12}$ **53.** $\log_3 \sqrt{x-2} = 2$ **54.** $2^{x+1} \cdot 8^{-x} = 4$

55. $8 = 4^{x^2} \cdot 2^{5x}$ **56.** $2^x \cdot 5 = 10^x$ **57.** $\log_6(x+3) + \log_6(x+4) = 1$

58. $\log_{10}(7x - 12) = 2\log_{10} x$ **59.** $e^{1-x} = 5$ **60.** $e^{1-2x} = 4$

61. $2^{3x} = 3^{2x+1}$ **62.** $2^{x^3} = 3^{x^2}$

In Problems 63 and 64, use the following result: If x is the atmospheric pressure (measured in millimeters of mercury), then the formula for the altitude $h(x)$ (measured in meters above sea level) is

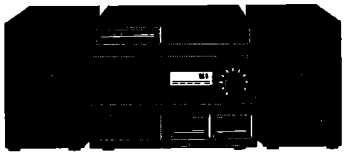
$$h(x) = (30T + 8000)\log\left(\frac{P_0}{x}\right)$$

where T is the temperature (in degrees Celsius) and P_0 is the atmospheric pressure at sea level, which is approximately 760 millimeters of mercury.

63. Finding the Altitude of an Airplane At what height is a Piper Cub whose instruments record an outside temperature of 0°C and a barometric pressure of 300 millimeters of mercury?

64. Finding the Height of a Mountain How high is a mountain if instruments placed on its peak record a temperature of 5°C and a barometric pressure of 500 millimeters of mercury?

65. Amplifying Sound An amplifier's power output P (in watts) is related to its decibel voltage gain d by the formula $P = 25e^{0.1d}$.



(a) Find the power output for a decibel voltage gain of 4 decibels.
(b) For a power output of 50 watts, what is the decibel voltage gain?

66. Limiting Magnitude of a Telescope A telescope is limited in its usefulness by the brightness of the star it is aimed at and by the diameter of its lens. One measure of a star's brightness is its *magnitude*: the dimmer the star, the larger its magnitude. A formula for the limiting magnitude L of a telescope, that is, the magnitude of the dimmest star that it can be used to view, is given by

$$L = 9 + 5.1\log d$$

where d is the diameter (in inches) of the lens.
(a) What is the limiting magnitude of a 3.5-inch telescope?
(b) What diameter is required to view a star of magnitude 14?

67. Salvage Value The number of years n for a piece of machinery to depreciate to a known salvage value can be found using the formula

$$n = \frac{\log s - \log i}{\log(1 - d)}$$

where s is the salvage value of the machinery, i is its initial value, and d is the annual rate of depreciation.
(a) How many years will it take for a piece of machinery to decline in value from \$90,000 to \$10,000 if the annual rate of depreciation is 0.20 (20%)?
(b) How many years will it take for a piece of machinery to lose half of its value if the annual rate of depreciation is 15%?

68. Funding a College Education A child's grandparents purchase a \$10,000 bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much will the bond fund be worth at maturity?

69. Funding a College Education A child's grandparents wish to purchase a bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much should they purchase so that the bond fund will be worth \$85,000 at maturity?