Exercises for Section 8

Prove that each equation is an identity.

1. $\cos t \tan t = \sin t$
2. $\sin t \cot t \sec t = 1$
3. $\dfrac{\sec t}{\csc t} = \tan t$
4. $\sin^2 \theta\,(1 + \cot^2 \theta) = 1$
5. $\tan^2 \theta = \dfrac{1}{\cos^2 \theta} - 1$
6. $(1 - \sin^2 \theta)(\sec^2 \theta) = 1$
7. $\dfrac{1}{\sec u + 1} - \dfrac{1}{\sec u - 1} = -2 \cot^2 u$
8. $\sec \alpha - \cos \alpha = \sin \alpha \tan \alpha$
9. $\dfrac{1}{1 + \cos u} + \dfrac{1}{1 - \cos u} = 2 \csc^2 u$
10. $\cot \alpha + \tan \alpha = \sec \alpha \csc \alpha$
11. $\dfrac{1 + \cos u}{\sin u} + \dfrac{\sin u}{1 + \cos u} = 2 \csc u$
12. $\dfrac{1 + \tan^2 \beta}{\tan^2 \beta} = \csc^2 \beta$
13. $\dfrac{1 - \sin^2 \beta}{\sin^2 \beta} = \cot^2 \beta$
14. $\dfrac{1 - \sin^3 \gamma}{1 - \sin \gamma} = 1 + \sin \gamma + \sin^2 \gamma$
15. $\sin \alpha - \csc \alpha = -\cos \alpha \cot \alpha$
16. $\dfrac{\tan^3 \gamma - \cot^3 \gamma}{\tan \gamma - \cot \gamma} = \sec^2 \gamma + \cot^2 \gamma$
17. $\dfrac{\sec^2 \gamma - 1}{\sec^2 \gamma} = \sin^2 \gamma$
18. $\dfrac{\sin^4 \gamma - \cos^4 \gamma}{\sin \gamma - \cos \gamma} = \sin \gamma + \cos \gamma$
19. $\dfrac{(1 + \cos A)(1 - \cos A)}{\sin A} = \sin A$
20. $\dfrac{\csc^2 A - \sin^2 A}{\csc A + \sin A} = \cos A \cot A$
21. $\dfrac{\sec^2 A - \tan^2 A}{\sin A} = \csc A$
22. $\sin(-x) \sec(-x) = -\tan x$
23. $\dfrac{\tan(-x)}{\sin x} = -\sec(-x)$
24. $\cot(-x) \sin(-x) = \cos(-x)$
25. $\dfrac{1 - \cos \alpha}{\sin \alpha} = \dfrac{\sin \alpha}{1 + \cos \alpha}$
26. $\dfrac{\sec \beta - 1}{\tan \beta} = \dfrac{\tan \beta}{\sec \beta + 1}$
27. $\csc \gamma - \cot \gamma = \dfrac{1}{\csc \gamma + \cot \gamma}$
28. $\dfrac{\tan t}{\tan t + \sec t} = \dfrac{1}{1 + \csc t}$
29. $\dfrac{\sec u}{\sec^2 u + \sec^3 u} = \dfrac{\cos u \cot u}{\cot u + \csc u}$
30. $\dfrac{\tan v \sec v - 1}{\tan v \sec v + \sec^2 v} = \dfrac{1 - \cot v \cos v}{1 + \csc v}$
31. $(1 - \cos t)^2 = 2 - 2 \cos t - \sin^2 t$
32. $(\sec t + \cos t)(\sec t - \cos t) = (\sec^4 t - 1)\cos^2 t$
33. $\sin^2 t(\sin t + \sec t)^2 = \sin^4 t + 2 \sin^3 t \sec t + \tan^2 t$
34. $\dfrac{\cos(-\alpha)}{1 + \tan \alpha} - \dfrac{\sin(-\alpha)}{1 - \tan \alpha} = \dfrac{\sec \alpha}{1 - \tan^2 \alpha}$
35. $\dfrac{1}{\sin(-\beta)} + \dfrac{1}{\csc(-\beta)} = -\sin \beta - \csc \beta$
36. $\cos(-\gamma) = \dfrac{2}{1 + \cos(-\gamma)} - \dfrac{1 - \cos \gamma + \sin^2 \gamma}{1 + \cos \gamma}$
37. $\dfrac{\sec A}{\cot^2 A} = \dfrac{1 - \cos^2 A}{\cos^3 A}$
38. $\dfrac{\sec B}{\sec B + \csc B} = \dfrac{1}{1 + \cot B}$
39. $\csc^4 C - \cot^4 C = \csc^2 C + \cot^2 C$
40. $\sin^3 \theta + \cos^3 \theta = (\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta)$
41. $\dfrac{1}{\tan \alpha + \cot \alpha} = \sin \alpha \cos \alpha$

sin, csc, tan, cot : odd $\Rightarrow -f(x) = f(-x)$

cos, sec : even $\Rightarrow f(x) = -f(x)$